

Some mixing properties of conditionally independent processes

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Motivation

The classical insurance risk model

$$R_n = u + c - S_n, \quad S_n = \sum_{i=1}^n X_i.$$

- In risk theory, some models treating the structure of dependence between individual risks X_i are factor models.
- In actuarial theory these models are known by models with common shock where dependence between r.v's X_i may come from a time-varying common factor.
- Many researchers in actuariat give attention to risk models in Markovian environment (Asmussen(1989) and Cossette *et al.*(2004)).
- An important property of factor models is: all risks have the property to be dependent but conditionally independent given the common factor.

Literature VS our Model

- Let V_i be the random factor observed at time i .
- All $i \in \mathbb{N}$, r.v.'s X_i are independent when conditioned to a factor $\underline{V}_n = (V_1, \dots, V_n)$.

Cossette *et al.* (2004) proposed a compound binomial model modulated by a Markovian environment. In their model, r.v.'s X_i are conditionally independent given r.v.'s V_i , roughly speaking

$$\mathbb{E}(f(X_{i_1}, \dots, X_{i_u}) | \underline{V}_n) = \int_{x_{i_1}, \dots, x_{i_u}} f(x_{i_1}, \dots, x_{i_u}) \prod_{j=i_1}^{i_u} N^j(dx_j, V_j)$$

In our framework, r.v.'s $(X_n)_{n \in \mathbb{N}}$ are considered to be conditionally independent given the entire trajectory of the factor:

$$\mathbb{E}(f(X_{i_1}, \dots, X_{i_u}) | \underline{V}_n) = \int_{x_{i_1}, \dots, x_{i_u}} f(x_{i_1}, \dots, x_{i_u}) \prod_{j=i_1}^{i_u} N^j(dx_j, \underline{V}_j),$$

where $N^j(dx_j, V_j)$ and $N^j(dx_j, \underline{V}_j)$ respectively denote the Kernel transition of X_j given V_j and given \underline{V}_j .

Goals

Our Goals are

- 1 To provide some mixing properties of the process $(X_i)_{i \in \mathbb{N}}$, under necessary conditions on the factor process $(V_i)_{i \in \mathbb{N}}$ and on the conditional mixing structure (see Definition 1).
- 2 To give a Self normalized Central Limit Theorem (SNCLT) for the sum $S_n = \sum_{i=1}^n X_i$.

Our framework is illustrated by some example in insurance.

Conditional independence

We shall make the following standing assumptions.

- H1: For all $i \in \mathbb{N}$, r.v.'s X_i are conditionally independent given $\underline{V}_i = (V_1, \dots, V_i)$.
- H2: The conditional law of $X_i | \underline{V}_n$ is the same as the one of $X_i | \underline{V}_i$, for all $n \geq i$.

Assumptions H1 and H2 are equivalent to:

For any multi index $i_1 \leq i_2 \dots \leq i_u$ and for any $n \geq i_u$ we have

$$\mathbb{E}(f(X_{i_1}, \dots, X_{i_u} | \underline{V}_n)) = \int_{x_{i_1}, \dots, x_{i_u}} f(x_{i_1}, \dots, x_{i_u}) \prod_{j=1}^u N^j(dx_{i_j}, \underline{V}_{i_j}), \quad (1.1)$$

where $N^j(dx_{i_j}, \underline{V}_{i_j})$ denotes the kernel transition of X_{i_j} given \underline{V}_{i_j} .

Conditional independence

Proposition 1

Let u, v and r be integers and let f and g be real valued bounded functions. Consider the multi-indices

$i_1 < i_2 \dots < i_u < i_u + r \leq j_1 < j_2 \dots < j_v$. If $H1$ and $H2$ hold then

$$\begin{aligned} & \text{Cov}(f(X_{i_1}, \dots, X_{i_u}), g(X_{j_1}, \dots, X_{j_v})) \\ &= \text{Cov}(\mathbb{E}(f(X_{i_1}, \dots, X_{i_u}) | \underline{V}_{i_u}), \mathbb{E}(g(X_{j_1}, \dots, X_{j_v}) | \underline{V}_{j_v})), \end{aligned} \quad (1.2)$$

This means that the covariance of $f(X_{i_1}, \dots, X_{i_u})$ and $g(X_{j_1}, \dots, X_{j_v})$ is equal to the covariance of their conditional expectation whenever conditioning is done respectively with respect to \underline{V}_{i_u} and \underline{V}_{j_v} .

Definition 1

Let u, v be integers, a random process (X_1, \dots, X_n) is said to be $\eta_{u,v}$ -mixing, if there exist a function $r \mapsto \eta(r)$ decreasing to 0 as r goes to infinity and a constant $C(u, v) > 0$ such that for any real valued bounded functions f and g , for any integers $u' \leq u, v' \leq v$ and for any multi-indices satisfying the relation (\star) :

$$i_1 < \dots < i_{u'} \leq i_u < i_u + r \leq j_1 < \dots < j_{v'} \leq j_v, \quad (\star)$$

we have

$$\sup |\text{Cov}(f(X_{i_1}, \dots, X_{i_{u'}}), g(X_{j_1}, \dots, X_{j_{v'}}))| \leq C(u, v) \eta(r) \|f\|_a \|g\|_b, \quad (1.3)$$

where the supremum is taken over all the sequences $(i_1, \dots, i_{u'})$ and $(j_1, \dots, j_{v'})$ satisfying (2.4) and $r \geq j_1 - i_u$ is the gap of time between past and future.

$\|\cdot\|_a$ and $\|\cdot\|_b$ are norms on bounded functions.

With respect to these norms, we have various kind of mixing as referred to the classical strongly mixing definitions.

- 1 If $\| \cdot \|_a = \| \cdot \|_b = \| \cdot \|_\infty$, we shall say that the process is $\alpha_{(u,v)}$ mixing and we shall write $\alpha(r)$ instead of $\eta(r)$. If the process is $\alpha_{(u,v)}$ for all u, v and if $\sup_{u,v \in \mathbb{N}} C(u, v) \leq C < \infty$ then the process is α mixing in the sense of Rosenblatt (1956).
- 2 If $\| \cdot \|_a = \| \cdot \|_b = \| \cdot \|_1$: the process is $\Psi_{(u,v)}$ mixing.

Note that

- $\eta(r)$ is decreasing in r ,
- for the **classical mixing coefficients**, $C(u, v)$ is uniform in u and v ,
- $\Psi_{(u,v)} \implies \alpha_{(u,v)}$ mixing.

Conditionally mixing sequences

Definition 2

Let u, v and r be integers and let (i_1, \dots, i_u) and (j_1, \dots, j_v) satisfying (2.4). The sequence of r.v.'s $(X_n)_{n \in \mathbb{N}}$ is called conditionally $\eta_{u,v}$ mixing with respect to \underline{V}_n if there exist a positive sequence $\eta(r) \rightarrow 0$ as $r \rightarrow \infty$ and a constant $C(u, v) > 0$ such that the following inequality holds for any real and bounded functions $f : \mathbb{R}^u \rightarrow \mathbb{R}$ and $g : \mathbb{R}^v \rightarrow \mathbb{R}$,

$$\begin{aligned} & \sup \left| \text{Cov}(\mathbb{E}(f(X_{i_1}, \dots, X_{i_u}) | \underline{V}_{i_{u'}}), \mathbb{E}(g(X_{j_1}, \dots, X_{j_v}) | \underline{V}_{j_{v'}})) \right| \\ & \leq C(u, v) \eta(r) \|f\|_a \|g\|_b, \end{aligned} \quad (1.4)$$

where the supremum is taken over all sequences (i_1, \dots, i_u) and (j_1, \dots, j_v) satisfying (2.4).

Some notations

Our main result emphasizes the relation between the conditional mixing properties of the process $(X_i)_{i \in \mathbb{N}}$, the mixing properties of the process $(V_i)_{i \in \mathbb{N}}$, and the regularity of the transition kernels. Recall that

$$N^{i_1, \dots, i_u}(\cdot, \underline{V}_{i_u}) = \prod_{k=1}^u N^{i_k}(\cdot, \underline{V}_{i_k})$$

where $N^{i_1, \dots, i_u}(\cdot, \underline{V}_{i_u})$ denote the kernel transition of the random vector $(X_{i_1}, \dots, X_{i_u})$ given $\underline{V}_{i_u} = (V_1, \dots, V_{i_u})$ and denote by

$$D(\underline{V}_{j_v}, \tilde{V}_{i_u, j_v, \ell}) = N^{j_1, \dots, j_v}(\cdot, \underline{V}_{j_v}) - N^{j_1, \dots, j_v}(\cdot, \tilde{V}_{i_u, j_v, \ell}),$$

where

- multi indices (i_1, \dots, i_u) and (j_1, \dots, j_v) , satisfying (\star) ,
- ℓ be an integer such that $0 < \ell \leq j_1 - i_u$,
- $\tilde{V}_{i_u, j_v, \ell} = (0, \dots, 0, V_{i_u + \ell + 1}, \dots, V_{j_1}, \dots, V_{j_v}) \in \mathbb{R}^{j_v}$.

Assumption 1

K_v and K'_v denote positive constants depending on v , $(\kappa(\ell))_{\ell \in \mathbb{N}}$ denotes a decreasing to zero sequence and $\tilde{N}_{j_1, \dots, j_v}(\cdot)$ is a positive finite measure on \mathbb{R}^v . We denote $k = j_1 - i_u - \ell$ with $0 < \ell \leq j_1 - i_u$. Let us introduce the following assumptions satisfied for multi indices satisfying (\star) .

$$\sup_{\underline{V}} \left| D(\underline{V}_{j_v}, \tilde{V}_{i_u, j_v, \ell}) \right| \leq K_v \|N^{j_1, \dots, j_v}(\cdot, \underline{V}_{j_v})\|_1 \kappa(k). \quad (2.1)$$

$$\sup_{\underline{V}} \left| D(\underline{V}_{j_v}, \tilde{V}_{i_u, j_v, \ell}) \right| \leq \tilde{N}_{j_1, \dots, j_v}(\cdot) \kappa(k). \quad (2.2)$$

$$\int \int_{\underline{X}_{j_1, \dots, j_v}} \left| D(\underline{V}_{j_v}, \tilde{V}_{i_u, j_v, \ell}) \right| \leq \kappa(k) K_v. \quad (2.3)$$

$$\|N^{j_1, \dots, j_v}(\cdot, \underline{V}_{j_v})\|_\infty \leq K'_v \|N^{j_1, \dots, j_v}(\cdot, \underline{V}_{j_v})\|_1. \quad (2.4)$$

$$\int \left\| N^{j_1, \dots, j_v}(\cdot, \underline{V}_{j_v}) \right\|_\infty < \infty. \quad (2.5)$$

X_{j_1, \dots, j_v}

Theorem 1

Assume $(X_i)_{i \in \mathbb{N}}$ is conditionally independent with respect to $(V_i)_{i \in \mathbb{N}}$.
 The results are presented in Table 1.

Table 1

Kind of mixing for $(V_i)_{i \in \mathbb{N}}$	Satisfied equation of assumption 1	Result: type of mixing for $(X_i)_{i \in \mathbb{N}}$
$\Psi_{u,v}$	(2.1)	$\Psi_{u,v}$
$\alpha_{u,v}$	(2.1) and (2.4)	$\Psi_{u,v}$
	(2.3) and (2.5)	$\alpha_{u,v}$

Proof of the main result

Lemma 1

Assume that $(V_i)_{i \in \mathbb{N}}$ is $\eta_{(u,v)}$ mixing in the sense of Definition 1. For any multi indices $i_1 < \dots < i_u < j_1 < \dots < j_v$ and $0 < \ell \leq j_1 - i_u$ and bounded functions f and g , we have

$$\begin{aligned} & \left| \text{Cov}(\mathbb{E}(f(X_{i_1}, \dots, X_{i_u}) | \underline{V}_{i_u}), \mathbb{E}(g(X_{j_1}, \dots, X_{j_v}) | \underline{V}_{j_v})) \right| \\ & \leq 2 \|f\|_1 \int_{x_{j_1}, \dots, x_{j_v}} |g| \cdot \sup_{\underline{V}} |D(\underline{V}_{j_v}, \tilde{V}_{i_u, j_v, \ell})| + \\ & \int_{x_{i_1}, \dots, x_{i_u}, x_{j_1}, \dots, x_{j_v}} |fg| \eta_{u,v}(\ell) \|N^{i_1, \dots, i_u}(\cdot, \underline{V}_{i_u})\|_a \cdot \|N^{j_1, \dots, j_v}(\cdot, \tilde{V}_{i_u, j_v, \ell})\|_b. \end{aligned}$$

Lemma 2

Assume that $(V_i)_{i \in \mathbb{N}}$ is $\eta_{(u,v)}$ mixing in the sense of Definition 1. For any multi indices $i_1 < \dots < i_u < j_1 < \dots < j_v$ and $0 < \ell \leq j_1 - i_u$ and bounded functions f and g , we have

$$\begin{aligned} & \left| \text{Cov}(\mathbb{E}(f(X_{i_1}, \dots, X_{i_u}) | \underline{V}_{i_u}), \mathbb{E}(g(X_{j_1}, \dots, X_{j_v}) | \underline{V}_{j_v})) \right| \\ & \leq 2 \int_{x_{i_1}, \dots, x_{i_u}} |f| \sup_{\underline{V}} N^{i_1, \dots, i_u}(\cdot, \underline{V}_{i_u}) \times \|g\|_\infty \int_{\underline{V}} \int_{x_{j_1}, \dots, x_{j_v}} |D(\underline{V}_{j_v}, \tilde{V}_{i_u, j_v, \ell})| \\ & + \int_{x_{i_1}, \dots, x_{i_u}, x_{j_1}, \dots, x_{j_v}} |f g| \eta_{u,v}(\ell) \|N^{i_1, \dots, i_u}(\cdot, \underline{V}_{i_u})\|_a \cdot \|N^{j_1, \dots, j_v}(\cdot, \tilde{V}_{i_u, j_v, \ell})\|_b. \end{aligned}$$

Note that when (2.1) or (2.2) are satisfied, we use Lemma 1 while when (2.3) is fulfilled, we use Lemma 2.

Proof of the Lemmas

$$\begin{aligned}
 & \left| \text{Cov}(\mathbb{E}(f(X_{i_1}, \dots, X_{i_u}) | \underline{V}_{i_u}), \mathbb{E}(g(X_{j_1}, \dots, X_{j_v}) | \underline{V}_{j_v})) \right| \\
 &= \left| \int_{x_{i_1}, \dots, x_{i_u}, x_{j_1}, \dots, x_{j_v}} f g \cdot \text{Cov}_{\underline{V}}(N^{i_1, \dots, i_u}(\cdot, \underline{V}_{i_u}), N^{j_1, \dots, j_v}(\cdot, \underline{V}_{j_v})) \right|,
 \end{aligned}$$

and

$$\begin{aligned}
 & \left| \text{Cov}_{\underline{V}}(N^{i_1, \dots, i_u}(\cdot, \underline{V}_{i_u}), N^{j_1, \dots, j_v}(\cdot, \underline{V}_{j_v})) \right| \\
 & \leq \left| \text{Cov}_{\underline{V}}(N^{i_1, \dots, i_u}(\cdot, \underline{V}_{i_u}), D(\underline{V}_{j_v}, \tilde{V}_{i_u, j_v, \ell})) \right| \\
 & + \left| \text{Cov}_{\underline{V}}(N^{i_1, \dots, i_u}(\cdot, \underline{V}_{i_u}), N^{j_1, \dots, j_v}(\cdot, \tilde{V}_{i_u, j_v, \ell})) \right|.
 \end{aligned}$$

Moment inequalities

Lemma 3

Let $(X_i)_{i \in \mathbb{N}}$ be a stationary $\alpha(u, v)$ mixing sequences. Let $\|f\|_p < \infty$ and $\|g\|_q < \infty$, where $1 < p, q \leq \infty$ and $\frac{1}{p} + \frac{1}{q} < 1$, then

$$\begin{aligned} & |\text{Cov}(f(X_{i_1}, \dots, X_{i_u}), g(X_{j_1}, \dots, X_{j_v}))| \\ & \leq (C(u, v) + 6)(\mathbb{E}|f|^p)^{\frac{1}{p}}(\mathbb{E}|g|^q)^{\frac{1}{q}}\alpha(r)^{1-\frac{1}{p}-\frac{1}{q}} \end{aligned}$$

where $r \geq j_1 - i_u$ where r is a positive integer.

Moment inequalities

Corollary 1

Let $S_n = \sum_{i=1}^n X_i$ where $E(X_i) = 0$ and denote by $\mathbb{E}(S_n^p)$ the moment of order p of the partial sum S_n . Let $\|X_n\|_r = \mathbb{E}(|X_n|^r)^{1/r}$, $\delta > 0$ and denote by

$$K_2 = \sup_i (C(1, 1) + 6) \|X_i\|_{2+\delta}^2, \quad K_4 = \sup_i (C(2, 2) + 6) \|X_i\|_{4+2\delta}^4.$$

Then

$$\mathbb{E}(S_n^2) \leq 2nK_2 \sum_{r=0}^{n-1} \alpha(r)^{\frac{\delta}{\delta+2}}, \quad (3.1)$$

$$\mathbb{E}(S_n^4) \leq 4!n^2K_2^2 \left(\sum_{r=0}^{n-1} \alpha(r)^{\frac{\delta}{\delta+2}} \right)^2 + nK_4 \sum_{r=0}^{n-1} (r+1)^2 \alpha(r)^{\frac{\delta}{\delta+2}}. \quad (3.2)$$

Moment inequalities

Theorem 2

Let $2 < p < r \leq \infty$, $2 < v \leq r$ and $X_n, n \geq 1$ be an α -mixing sequence of random variables with $\mathbb{E} X_n = 0$. Assume that for some $C > 0$ and $\theta > 0$

$$\alpha(n) \leq Cn^{-\theta}, \quad (3.3)$$

where $\theta > \frac{pr}{(2(r-p))}$. Then, there exists $A = A(\varepsilon, r, p, \theta, C) < \infty$ for any $\varepsilon > 0$, such that

$$E|S_n|^p \leq An^{p/2} \left((K_2 D_{n,2,2})^{p/2} + \max_{i \leq n} \|X_i\|_r^p \right), \quad (3.4)$$

Self Normalised Central Limit Theorem

We introduce the following estimator which is a modified version of some estimators introduced in Shao *et al.*(1995)(write $\ell = \ell_n$).

$$B_{n,2}^2 = \frac{1}{n - \ell + 1} \sum_{j=0}^{n-\ell} \left(\frac{S_j(\ell) - \bar{X}_\ell}{\sqrt{\ell}} \right)^2,$$

with

$$S_j(\ell) = \sum_{k=j+1}^{j+\ell} X_k \text{ and } \bar{X}_\ell = \frac{1}{n - \ell + 1} \sum_{j=0}^{n-\ell} S_j(\ell).$$

Self Normalised Central Limit Theorem

Theorem 3

Let $\{X_n, n \in \mathbb{N}\}$ be a stationary $\alpha_{(1,1)}$ mixing process with $\mathbb{E}(X_1) = \mu$.
 Assume that for some $\delta > 0$,

$$\mathbb{E}|X_1|^{2+\delta} < \infty, \quad (3.5)$$

$$\frac{\text{Var}(S_n)}{n} \rightarrow \sigma^2, \text{ as } n \rightarrow \infty, \quad (3.6)$$

$$\sum_{r=1}^{\infty} \alpha(r)^{\frac{\delta}{2+\delta}} < \infty, \quad (3.7)$$

$$l_n = l, l_n \rightarrow \infty, l = o(n), n \rightarrow \infty \quad (3.8)$$

If $0 < \sigma < \infty$, then $B_{n,2}$ is a consistent estimator of σ in the L^2 -norm,

$$\frac{S_n - n\mu}{\sqrt{nB_{n,2}}} \xrightarrow{d} N(0,1). \quad (3.9)$$

Self Normalised Central Limit Theorem

Theorem 4

Let $\{X_n, n \in \mathbb{N}\}$ be a stationary $\alpha_{(2,2)}$ mixing process satisfying mixing condition with $\mathbb{E}(X_1) = \mu$. Assume that

$$\left\{ \left(\frac{S_n - n\mu}{\sqrt{n}} \right)^{4+2\delta}, n \geq 1 \right\}, \quad (3.10)$$

is uniformly integrable with $\delta > 0$. Assume that $\ell_n = \ell$, $\ell_n \rightarrow \infty$, $\ell = o(n)$ as $n \rightarrow \infty$. If in addition 3.7 and for $0 < \sigma < \infty$, 3.6 holds then

$$\frac{\sqrt{n}}{\ell} \left(B_{n,2} - \mathbb{E} \left(\left| \frac{S(\ell) - \ell\mu}{\sqrt{\ell}} \right|^2 \right)^{\frac{1}{2}} \right) \xrightarrow{d} N(0, \frac{\sigma^2}{3}). \quad (3.11)$$

Example in a discrete case

Consider the process $(X_i)_{(i \in \mathbb{N})}$ such that $X_i = I_i \times B_i$, where

- the I_i 's are Bernoulli r.v.'s, conditionally independent with respect to \underline{V}_i ,
- the claim amounts B_i 's are considered independent and independent of the I_i 's and of \underline{V}_i .
- $(V_i)_{(i \in \mathbb{N})}$ is mixing sequence of Bernoulli random variables taking value one in time of crisis and value zero in a stable period.

We shall consider that the conditional Law of I_i has the following structure:

$$P(I_i = 1 | \underline{V}_i) = \sum_{j=1}^i \frac{h(V_j)}{2^{i-j}},$$

where h is a measurable, non negative and bounded function:

$0 < \rho \leq h(v) \leq \frac{\kappa}{2}$, with $\kappa < 1$ to ensure that $P(I_i = 1 | \underline{V}_i) < 1$.

Lemma 3

The transition kernel $N^{i_1, \dots, i_u}(\cdot | \underline{V}_{i_u})$ satisfies conditions (2.1) and (2.4) of Assumption 1.

where we recall

$$\sup_{\underline{V}} \left| D(\underline{V}_{j_v}, \tilde{V}_{i_u, j_v, \ell}) \right| \leq K_v \|N^{j_1, \dots, j_v}(\cdot, \underline{V}_{j_v})\|_1 \kappa(k), \quad (2.1)$$

$$\|N^{j_1, \dots, j_v}(\cdot, \underline{V}_{j_v})\|_\infty \leq K'_v \|N^{j_1, \dots, j_v}(\cdot, \underline{V}_{j_v})\|_1. \quad (2.4)$$

Estimation of the variance in the discrete example

Let $S_n = \sum_{i=1}^n I_i$.

$$P(I_i = 1 | \underline{V}_i) = K \sum_{j=1}^i \frac{1 + V_j}{2^{i-j}},$$

The exact value of the limit variance is derived equal to

$$\lim_{n \rightarrow \infty} \frac{\text{Var}(S_n)}{n} = \frac{8}{3} K^2 q (1 - q) + 2K(1 + q - 2K - 4Kq - 2K^2 q^2)$$

Numerical application: Let $K = 0.24$ and $q = 0.6$ then the exact value of the squared error σ is equal to 0.4637. We also derive easily the squared error when the conditioning is with respect to only the last value of the factor, and the result is equal to $\sigma = 0.4864$.

Simulations

For simulation we choose $\ell = \sqrt{n}$ and results for $B_{n,2}$ are given in the following table

Table 2

n	m	$B_{n,\ell}$	IC-	IC+	EQMA	EQMR
200000	50	0,4607	0,4360	0,4854	0,000148	0,000689
	100	0,4621	0,4374	0,4869	0,000135	0,000627
50000	300	0,4615	0,4266	0,4964	0,000261	0,001213
	400	0,4624	0,4274	0,4974	0,000258	0,001198

Example in continuous case

The conditional kernel transition is denoted by $N^i(dx_i, \underline{V}_i) = f_{\underline{V}_i}^i(\cdot)dx_i$.

Example 1

Consider that the conditional law $X_i | \underline{V}_i$ is $\text{Pareto}(\alpha, \theta_i)$ where $\alpha > 2$ is the shape parameter and $\theta_i > 0$ is the scale parameter: the conditional density of X_i , $i \in \mathbb{N}$, has the form

$$f_{\underline{V}_i}^i(x_i; \alpha, \theta_i) = \alpha \times \frac{\theta_i^\alpha}{x_i^{\alpha+1}} \text{ for } x_i \geq \theta_i,$$

where the scale parameter θ_i depends on \underline{V}_i with the following structure:

$$\theta_i^\alpha = \sum_{j=1}^i \frac{h(V_j)}{2^{i-j}}, \quad (4.1)$$

and where the function h satisfies $0 < h(v) \leq \tau_2$.

Proposition 2

If $0 < \tau_1 \leq h(v) \leq \tau_2$ for any v then the process of Example 1 satisfies (2.3) and (2.5) of Assumption 1.

Remark

Note that the case of an Exponential conditional law (instead of a Pareto conditional law) could also be considered. In particular if we consider an Exponential law with parameter $(1/\theta_i)$ where θ_i is defined as in (4.1) with $\alpha = 1$ and $0 < \tau_1 \leq h(v) \leq \tau_2$ for any v , then (2.3) and (2.5) of Assumption 1 are satisfied.

Models for applications of the absolutely continuous case

Consider a hedging strategy in an insurance company.

Assume that $(Z_i)_{i \in \mathbb{N}}$ is a sequence of i.i.d. r.v.'s representing random transaction costs of the hedging strategy that are observed in a steady environment.

Assume that the $(V_j)_{j \geq 1}$ are independent of Z_i and are r.v.'s which contribute to create liquidity shocks and market conditions during some time.

Consider for each $i \geq 1$, $X_i = \theta_i Z_i$ where








$$\theta_i = \left(\sum_{j=1}^i \frac{h(V_j)}{2^{i-j}} \right)^{1/\alpha}$$

is a multiplicative factor, the function h is as in the previous example.

If Z_i is Pareto $(\alpha, 1)$ distributed, then X_i is Pareto (α, θ_i) distributed as in the previous Example.

Conclusion

- 1 We have proved some mixing properties for process $(X_i)_{i \in \mathbb{N}}$ when r.v.'s are considered to be conditionally independent given an unbounded memory of a factor.
- 2 We proved a Self Normalised Central Limit Theorem for $\sum_{i=1}^n X_i$ adapted with mixing properties satisfied by our models, namely $\eta_{u,v}$ mixing properties.
- 3 We give some illustrated examples and we compare the aggregate risk of an insurance portfolio of conditionally independent risks when the conditioning is done with respect to a bounded memory of the factor or with respect to unbounded memory and hence obtain additional information about the magnitude of risk.

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Thank you for your attention.