A VERY STRANGE INVARIANT MEASURE

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In recent work (Höpfner, Hoffmann, Löcherbach, to appear in SJS 2002) onnonparametric estimation of an unknown branching rate from time continuousobservation of an ergodic branching diffusion (a particle process whereparticles travel on diffusion paths, die at position dependent rates and leaveoffspring generated according to position dependent reproduction laws, with animmigration component) in dimension $d = 1$ the main problem turned out to beable to control the Lebesgue density of the invariant measure $\bar{m}$ on$R$ and to specify its smoothness properties. For branching particle systems with interaction between particles of aconfiguration, we can prove in dimension $d = 1$ - the invariant density on R is continuous, - the invariant density on R coincides with the invariant density of a branching diffusion whose drift and local mass reduction rate are obtained from the corresponding rates of the original interacting process by 'palming out'. Here 'palming out' means integrating out the configuration dependence, using aconditional version - given that a site $a \in R$ is occupied - of theinvariant law m for the particle process $\varphi = (\varphi_t)_{t\geq 0}$on the configuration space S, the set of all finite particleconfigurations. In order to do this, one has to know the invariant law m on S. For generaldimension d, we give a representation for m and prove that m admits -for arbitrary dimension d - a Lebesgue density. However, this density is farfrom being continuous on S since it may (and will) take the value $+\infty$on a variety of hyperplanes in S. The best result which one can hope for is smoothness of the density inrestriction to the open set $D \subset S$ of configurations without 'doublepoints' (all particles sitting in different positions). We report on work inprogress to prove this, and on representations of the density of the invariantmeasure $\bar{m}$ on the single particle state space R^d.