Mixed fractional Brownian motion: the filtering perspective

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7 October 2013 / Le Mans
Mixed fBm: the filtering perspective

Cai, Chigansky, Kleptsyna

Problems statement and questions around

Historical survey

Main Tools

The Main Results

Stochastic Analysis

Semimartingale Structure of X

Drift estimation in mixed fractional noise

Auxiliary Results

Mixed fBm for $H < 1/2$

Integro-Differential Equation

The proofs

Diffusion type representation, Equivalence of measures

large sample asymptotic of MLE

Mixed fBm for $H > 1/2$
Objects of studies

\[ X_t = B_t + B_t^H \]

where

- \( B_t \) — is the standard Brownian motion
- \( B_t^H \) — is an independent fractional Brownian motion
- \( H \in (0, 1] \) — is known Hurst parameter

\[ Y_t = \int_0^t f(s) \, ds + X_t, \quad 0 \leq t \leq T. \]
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fractional Brownian motion

**Fractional Brownian motion**

$B_t^H$ is a fractional Brownian motion with Hurst parameter $H \in (0, 1)$, i.e. zero mean Gaussian process with the correlation function

$$K(s, t) = \mathbb{E} B_t^H(t) B_s^H(s) = \frac{1}{2} \left( |t|^{2H} + |s|^{2H} - |t - s|^{2H} \right),$$

for $s, t \in [0, T].$

$B_t^H$ is not a semimartingale on its own filtration, unless $H = \frac{1}{2}$ or $H = 1.$
fractional Brownian motion

$B^H_t$ is a fractional Brownian motion with Hurst parameter $H \in (0, 1)$, i.e. zero mean Gaussian process with the correlation function

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$B^H$ is not a semimartingale on its own filtration, unless $H = \frac{1}{2}$ or $H = 1.$
Questions for discussions

- Stochastic analysis of $X$:
  - structure of the fundamental martingale
  - the semimartingale representation
  - the density with respect to the standard and fractional Wiener measures

- Stochastic analysis of $Y$: the density with respect to measure $\mu^X$

- Potential applications; MLE asymptotic properties
2000 Patric Cheridito

\[
\sup_{\tau} \sum_{j=0}^{n-1} \mathbb{E} \left| \mathbb{E} (X_{t_{j+1}} - X_{t_{j}} \mid \mathcal{F}_{t_{j}}^{X}) \right| < \infty
\]

2003 Fabrice Boduin and David Nualart

\[X := B + V, \quad \partial^{2}K/\partial s\partial t \in L^{2}([0, T]^{2})\]

Hida-Hitsuda criterion.

2007 Harry van Zanten equivalence of \(\xi = \sum_{k=1}^{n} \alpha_{k} B^{H_{k}}\) of \(n\) independent fBm's to a single fBm. (Spectral techniques for processes with stationary increments).
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\[
\sup_{\tau} \sum_{j=0}^{n-1} \mathbb{E} \left| \mathbb{E}(X_{t_{j+1}} - X_{t_j} \| \mathcal{F}_t^X) \right| < \infty
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of \( n \) independent fBm's to a single fBm. (Spectral techniques for processes with stationary increments).
Main Tools

\[ \mathbb{F}^X = (\mathbb{F}_t^X) \text{ and } \mathbb{F} = (\mathbb{F}_t), \ t \in [0, T] \] —the natural filtrations of \( X \) and \( (B, B^H) \) respectively.

Fundamental Martingale

\[ M_t = \mathbb{E}(B_t | \mathbb{F}_t^X), \quad t \in [0, T]. \]

\( M \) encodes many of the essential features of the process \( X \), making its structure particularly transparent.
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Auxiliary Results

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Fundamental Martingale Representation via $X$

**Mathematical Formulation**

\[ M_t = \int_0^t g(s, t) dX_s, \quad \langle M \rangle_t = \int_0^t g(s, t) ds, \quad t \geq 0. \]

The kernel $g(s, t)$ solves integro-differential equation:

**Equation for the Kernel**

\[ g(s, t) + H \frac{d}{ds} \int_0^t g(r, t) |s-r|^{2H-1} \text{sign}(s-r) dr = 1, \quad 0 < s < t \leq T. \]

The family of functions \( \{g(s, t), 0 \leq s \leq t \leq T\} \) plays the key role in our approach to analysis of the mixed fBm.
Fundamental Martingale Representation via $X$

**Fundamental Martingale Representation**

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The family of functions $\{g(s, t), 0 \leq s \leq t \leq T\}$ plays the key role in our approach to analysis of the mixed fBm.
**X is a stochastic integral w.r.t M**

<table>
<thead>
<tr>
<th>Representation of X via M</th>
</tr>
</thead>
<tbody>
<tr>
<td>The following representation holds:</td>
</tr>
<tr>
<td>[ X_t = \int_0^t G(s, t) dM_s, \quad t \in [0, T], ]</td>
</tr>
</tbody>
</table>

where

\[ G(s, t) := 1 - \frac{d}{d\langle M \rangle_s} \int_0^t g(\tau, s) d\tau, \quad 0 \leq s \leq t \leq T. \]

and, in particular, \( \mathbb{F}_t^X = \mathbb{F}_t^M \), \( P \)-a.s. for all \( t \in [0, T] \).
The fundamental Semimartingale

Let $Y = (Y_t)$ defined by

$$Y_t = \int_0^t f(s) ds + X_t, \quad t \in [0, T],$$

Then $Y$ admits the representation

$$Y_t = \int_0^t G(s, t) dZ_s$$

The fundamental semimartingale $Z = (Z_t)$

$$Z_t = \int_0^t g(s, t) dY_s = M_t + \int_0^t \Phi(s) d\langle M \rangle_s,$$

and

$$\Phi(t) = \frac{d}{d\langle M \rangle_t} \int_0^t g(s, t) f(s) ds.$$
The measures \( \mu^X \) and \( \mu^Y \)

In particular, \( \mathbb{F}^Y_t = \mathbb{F}^Z_t \), \( P \)-a.s. for all \( t \in [0, T] \) and, if

\[
E \exp \left\{ - \int_0^T \Phi(t) \, dM_t - \frac{1}{2} \int_0^T \Phi^2(t) \, d\langle M \rangle_t \right\} = 1,
\]

then the measures \( \mu^X \) and \( \mu^Y \) are equivalent and the corresponding Radon-Nikodym density is given by

\[
\frac{d\mu^Y}{d\mu^X}(Y) = \exp \left\{ \int_0^T \hat{\Phi}(t) \, dZ_t - \frac{1}{2} \int_0^T \hat{\Phi}^2(t) \, d\langle M \rangle_t \right\},
\]

where \( \hat{\Phi}(t) = E(\Phi(t)|\mathbb{F}^Y_t) \).
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Let $H \in \left(\frac{3}{4}, 1\right]$. Then $X$ is a **diffusion** type process:

$$X_t = W_t - \int_0^t \varphi_s(X)ds, \quad W_t = \int_0^t \frac{dM_s}{g(s, s)},$$

$W$ is an $F^X$-Brownian motion; $\varphi_t(X) = \int_0^t \frac{\dot{g}(s, t)}{g(t, t)} dX_s$
Moreover, the measures $\mu^X$ and $\mu^W$ are equivalent and

$$\frac{d\mu^X}{d\mu^W}(X) = \exp \left\{ - \int_0^T \varphi_t(X) dX_t - \frac{1}{2} \int_0^T \varphi_t^2(X) dt \right\}.$$
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Drift estimation in mixed fractional noise

Let

\[ Y_t = \theta t + B_t + B^H_t, \quad t \in [0, T] \]

**MLE of \( \theta \)**

The MLE of \( \theta \) is given by

\[
\hat{\theta}_T(Y) = \frac{\int_0^T g(s, T)dY_s}{\int_0^T g(s, T)ds},
\]

For \( H \in (0, 1) \) this estimator is strongly consistent and the corresponding estimation error is normal

\[
\hat{\theta}_T - \theta \sim N\left(0, \frac{1}{\int_0^T g(s, T)ds}\right).
\]
Drift estimation in mixed fractional noise

### Asymptotic variance

With the following asymptotic variance:

- for $H > \frac{1}{2}$,

\[
\lim_{T \to \infty} T^{2-2H} \mathbb{E}(\hat{\theta}_T - \theta)^2 = \frac{2H \Gamma(H + \frac{1}{2}) \Gamma(3 - 2H)}{\Gamma(\frac{3}{2} - H)},
\]

where $\Gamma(\cdot)$ is the standard Gamma function.

- for $H < \frac{1}{2}$,

\[
\lim_{T \to \infty} T \mathbb{E}(\hat{\theta}_T - \theta)^2 = 1.
\]
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Indirect approach

Reduction to the case $H > 1/2$

The trick is to transform $X$ into

$$\tilde{X}_t = \int_0^t \tilde{\rho}(s, t) dX_s, \quad t \in [0, T],$$

where the kernel $\tilde{\rho}(s, t)$ is such that the process

$$\tilde{B}_t = \int_0^t \tilde{\rho}(s, t) dB_s^H,$$

is a standard Brownian motion.
Indirect approach, II

Reduction to the case $H > 1/2$

Then the Gaussian process

$$\tilde{U}_t = \int_0^t \tilde{\rho}(s, t) dB_s$$

has covariance function with integrable partial derivative

$$\tilde{\kappa}(s, t) := \frac{\partial^2}{\partial s \partial t} E \tilde{U}_s \tilde{U}_t = |t - s|^{-2H} \chi \left( \frac{s \land t}{s \lor t} \right), \quad s \neq t,$$

where $\chi(\cdot)$ is a continuous function, and the process $\tilde{X} = \tilde{B} + \tilde{U}$ with $H < \frac{1}{2}$ has the structure, similar to the original process $X$ with $H > \frac{1}{2}$. 
Reduction to the case $H > 1/2$

Then the Gaussian process

$$\widetilde{U}_t = \int_0^t \widetilde{\rho}(s, t) dB_s$$

has covariance function with integrable partial derivative

$$\widetilde{\kappa}(s, t) := \frac{\partial^2}{\partial s \partial t} \mathbb{E} \widetilde{U}_s \widetilde{U}_t = |t - s|^{-2H} \chi \left( \frac{s \wedge t}{s \vee t} \right), \quad s \neq t,$$

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The kernel \( g(s, t) \) is the unique continuous solution of the following equations:

- for \( H \in (0, 1] \), the integro-differential equation:
  \[
  g(s, t) + H \frac{d}{ds} \int_0^t g(r, t) |s - r|^{2H-1} \text{sign}(s - r) \, dr = 1.
  \]

- for \( H \in (\frac{1}{2}, 1] \), the weakly singular integral equation:
  \[
  g(s, t) + H(2H - 1) \int_0^t g(r, t) |s - r|^{2H-2} \, dr = 1.
  \]
Alternative Forms, II

- for $H \in (0, 1)$, the **fractional** integro-differential equation

\[
c_H \frac{d}{ds} \int_0^s g(r, t) r^{1/2-H} (s - r)^{1/2-H} \, dr -
\]

\[
- \alpha_H s^{1-2H} \frac{d}{ds} \int_s^t g(r, t) r^{H-1/2} (r - s)^{H-1/2} \, dr
\]

\[
= c_H s^{1/2-H} (t - s)^{1/2-H}.
\]
Alternative Forms, III

For \( H \in (0, \frac{1}{2}) \), the weakly singular integral equation

\[
g(s, t) + \beta_H t^{-2H} \int_0^t g(r, t) \kappa \left( \frac{r}{t}, \frac{s}{t} \right) dr = c_H s^{1/2-H} (t-s)^{1/2-H},
\]

with the kernel

\[
\kappa(u, v) = (uv)^{1/2-H} \int_{u \vee v}^1 r^{2H-1} (r-u)^{-1/2-H} (r-v)^{-1/2-H} dr
\]
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Integral Equation with $H > 1/2$, I

Properties of $g(s, t)$ on the diagonal

The function $g(t, t)$, $t \in [0, T]$ satisfies the properties:

- $g(t, t)$ is continuous on $[0, T]$ with $g(0, 0) := \lim_{t \to 0} g(t, t) = 1$
- $g(t, t) > 0$ for all $t \in [0, T]$

$$\int_0^t g(s, t) ds = \int_0^t g^2(s, s) ds.$$
Integral Equation with $H > 1/2$, II

Properties of $\dot{g}(s, t) = \frac{\partial}{\partial t} g(s, t)$

The kernel $g(s, t)$ satisfies the following properties

- $g(s, t)$ is continuously differentiable at $t \in (0, T]$ for any $s > 0$, $s \neq t$;
- the derivative $\dot{g}(s, t) := \frac{\partial}{\partial t} g(s, t)$ satisfies the equation

$$\dot{g}(s, t) + H(2H - 1) \int_0^t \dot{g}(r, t)|r - s|^{2H-2} dr =$$

$$-H(2H - 1)g(t, t)|r - s|^{2H-2}, s \in (0, t).$$

- $\dot{g}(\cdot, t) \in L^2([0, t])$ for $H > 3/4$
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Integro-Differential Equation

$H > 3/4$

**$X$ is a diffusion type process**

\[
M_t = \int_0^t g(s, t) dX_s = \int_0^t g(s, s) dX_s + \int_0^t \left( g(r, t) - g(r, r) \right) dX_r
\]

\[
\int_0^t g(s, s) dX_s + \int_0^t \int_r^t \dot{g}(r, s) dsdX_r =
\]

\[
\int_0^t g(s, s) dX_s + \int_0^t \int_0^s \dot{g}(r, s) dX_r ds,
\]

where the last equality holds since $\dot{g}(\cdot, s) \in L^2([0, s])$
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$H > 3/4$, II

$X$ is a diffusion type process, II

Hence

$$W_t = \int_0^t \frac{1}{g(s, s)} dM_s = X_t + \int_0^t \int_0^s \frac{g(r, s)}{g(s, s)} dX_r ds =:$$

$$X_t + \int_0^t \varphi_s(X) ds.$$
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$H \leq 3/4$

Singularity of Measures

\[ L_t := \int_0^t g(s, t) dW_s, \]

must be a semimartingale under $P$. Define

\[ \psi(s, t) = -\int_s^t g(r, r) \sum_{m=1}^{n_0-1} (-1)^m \kappa^{(m)}(r, s) dr, \quad 0 < s < t \leq T, \]

where $n_0$ is the least integer greater than $\frac{1}{4H-2}$. Then

$\psi(\cdot, t) \in L^2([0, t])$. 
Singularity of Measures, II

Define

\[ U_t := \int_0^t \psi(s, t) dW_s \]

\[ V_t := \int_0^t (g(s, t) - g(s, s) + \psi(s, t)) dW_s. \]

Then

\[ \int_0^t g(s, t) dW_s = V_t + \int_0^t g(s, s) dW_s - U_t. \]

- \( U \) has zero quadratic variation, but unbounded first variation
- \( V \) has bounded first variation.
Singular perturbations

Fix $\varepsilon > 0$ and let $g_\varepsilon$ be the solution of the equation:

$$\varepsilon g_\varepsilon(\varphi)(u) + \int_0^1 g_\varepsilon(\varphi)(v) \kappa(u,v) dv = \varphi(u), \quad u \in [0, 1],$$

where $\varphi$ is a sufficiently smooth function. Let $g(\varphi)$ be the solution of auxiliary integral equation of the first kind

$$\int_0^1 g(\varphi)(v) \kappa(u,v) dv = \varphi(u).$$
Singular perturbations

Let $\psi(u)$ be a function, such that $g^{(\psi)}$ exists, then

$$\left| \int_0^1 (g^{(\varphi)}(s) - g^{(\varphi)}(s)) \psi(s) \, ds \right| \leq$$

$$2\varepsilon \left( \int_0^1 (g^{(\psi)}(u))^2 \, du \right)^{1/2} \left( \int_0^1 (g^{(\varphi)}(u))^2 \, du \right)^{1/2}. $$

Singular perturbations

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Mixed fBm for $H < \frac{1}{2}$

Integro-Differential Equation
Concluding Remarks

Open Questions

- Pathways convergence for a singulary perturbed equations
- Boundary layer construction