Nonsynchronous Covariation with Application to High-Frequency Finance

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SAPS VI, Université du Maine

March 22, 2007

OUTLINE

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PART I. Motivation and Background

- (X^1, X^2): (log-)price processes,

$$dX_t^k = \mu_t^k dt + \sigma_t^k dW_t^k, \ \ k = 1, 2,$$

  with ρ_t correlation of W^1 and W^2.
- $(X_{t_i}^1, X_{t_i}^2)_{i=0,1,\ldots,m}$: samples at $\Pi := \{0 = t_0, t_1, \cdots, t_m = T\}$.
- Quantity of interest:

$$\left[X^1, X^2\right]_T = \int_0^T \sigma_t^1 \sigma_t^2 \rho_t dt.$$

- **Realized covariance**:

$$RCV\left(X^1, X^2; \Pi\right) := \sum_{i=1}^{m} (X_{t_i}^1 - X_{t_{i-1}}^1)(X_{t_i}^2 - X_{t_{i-1}}^2). \qquad (1)$$

$$RCV\left(X^1, X^2; \Pi\right) \;\; \stackrel{P}{\rightarrow} \;\; \left[X^1, X^2\right]_T = \int_0^T \sigma_t^1 \sigma_t^2 \rho_t dt,$$

as $\pi(m) := \max_{1 \leq i \leq m} |t_i - t_{i-1}| \rightarrow 0$.

"consistency"

- **Realized volatility**: case $X^1 \equiv X^2$:

$$RV\left(X^1; \Pi\right) := RCV\left(X^1, X^1; \Pi\right)$$

Realized correlation:

$$R_{\Pi}^{1,2} := \frac{RCV\left(X^1, X^2; \Pi\right)}{\sqrt{RV\left(X^1; \Pi\right)}\sqrt{RV\left(X^2; \Pi\right)}}$$
$$= \frac{\sum_{i=1}^{m}(X_{t_i}^1 - X_{t_{i-1}}^1)(X_{t_i}^2 - X_{t_{i-1}}^2)}{\sqrt{\sum_{i=1}^{m}(X_{t_i}^2 - X_{t_{i-1}}^2)^2}\sqrt{\sum_{i=1}^{m}(X_{t_i}^2 - X_{t_{i-1}}^2)^2}}. \tag{2}$$

Realized beta (or regression):

$$b_{\Pi}^{1\sim 2} := \frac{\sum_{i=1}^{m}(X_{t_i}^1 - X_{t_{i-1}}^1)(X_{t_i}^2 - X_{t_{i-1}}^2)}{\sum_{i=1}^{m}(X_{t_i}^2 - X_{t_{i-1}}^2)^2}. \tag{3}$$

-Andersen and Bollerslev (98), *Int'l Economic Rev.*

-Comte and Renault (98), *MF*

-Andersen, Bollerslev, Diebold, Ebens (01), *JFE*	ABDE
-Andersen, Bollerslev, Diebold, Labys (01), *JASA*	ABDL
-Barndorff-Nielsen and Shephard (02), *JRSS-B*	BN-S(02)
-Barndorff-Nielsen and Shephard (04), *Econometrica*	BN-S(04)

Tick-by-tick data

Citigroup (C):

C	12/23/2002	N	9:30:23	37.65000000	341200		0	3364881	40
C	12/23/2002	N	9:30:29	37.65000000	3700		0	3364882	40
C	12/23/2002	N	9:30:31	37.65000000	4100		0	3364883	40
C	12/23/2002	N	9:30:38	37.63000000	500	E	0	3364885	40
C	12/23/2002	N	9:30:38	37.63000000	800	E	0	3364887	40
C	12/23/2002	N	9:30:39	37.63000000	200		0	3364889	40
C	12/23/2002	N	9:30:52	37.63000000	2500		0	3364894	40
C	12/23/2002	N	9:30:57	37.63000000	6200		0	3364897	40
C	12/23/2002	N	9:31:01	37.61000000	1400		0	3364898	40
C	12/23/2002	N	9:31:07	37.60000000	55000		0	3364902	40

....

The Trade And Quote (TAQ) database of NYSE: www.nyse.com/taq

Data nonsynchronicity

Since tick data are *irregularly* spaced, realized covariance requires to covert them into *regularly* spaced.

- Regular interval size: h (min) $\qquad h = T/m$
- Data interpolation: 2 common schemes
  - **previous tick interpolation**

    uses the most recent values
  - **linear interpolation**

    uses observations bracketing the desired time

“**synchronization**”

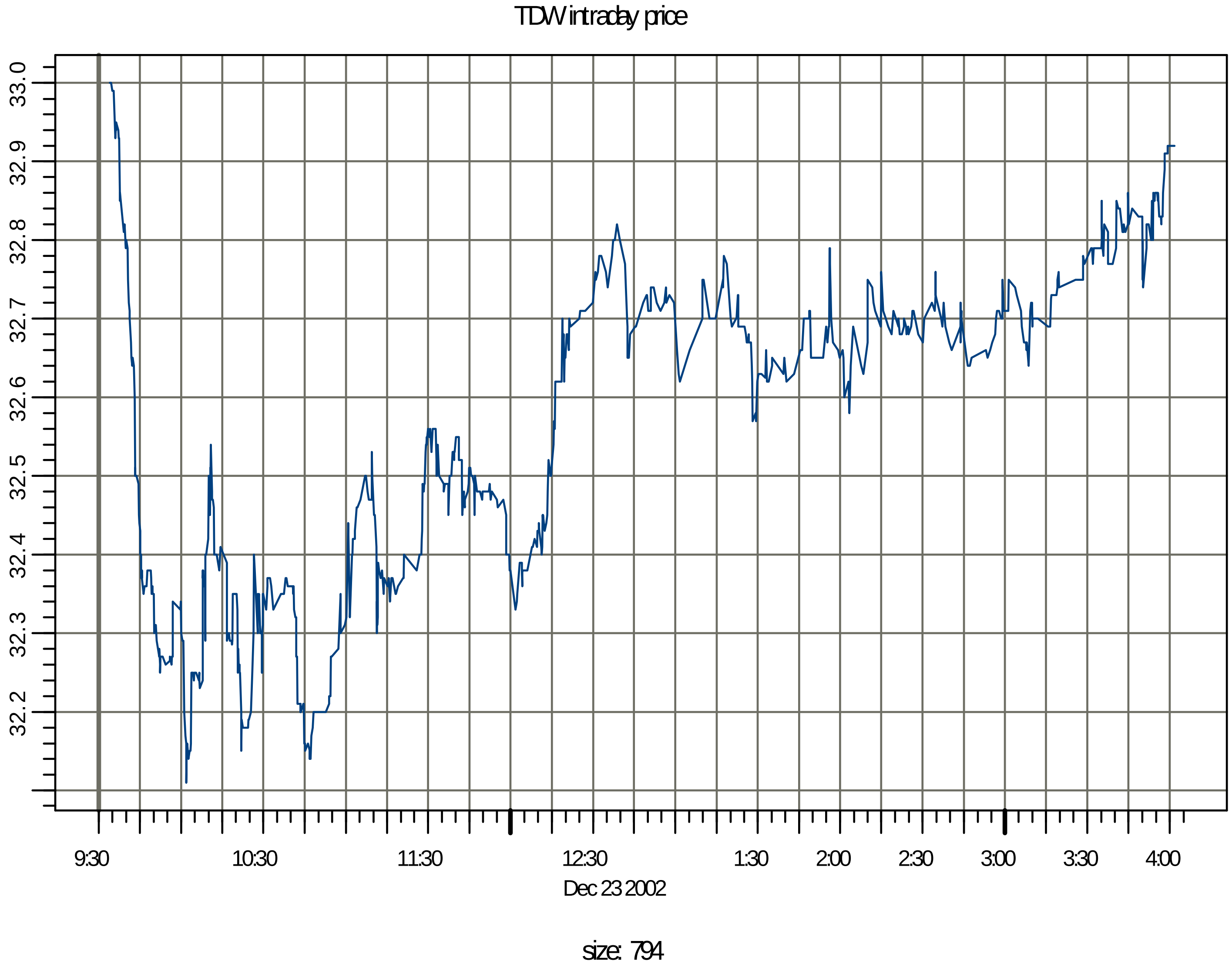
Common practice:

- $h = 5$ (ABDE(01), ABDL(01)) ~ 30 min.

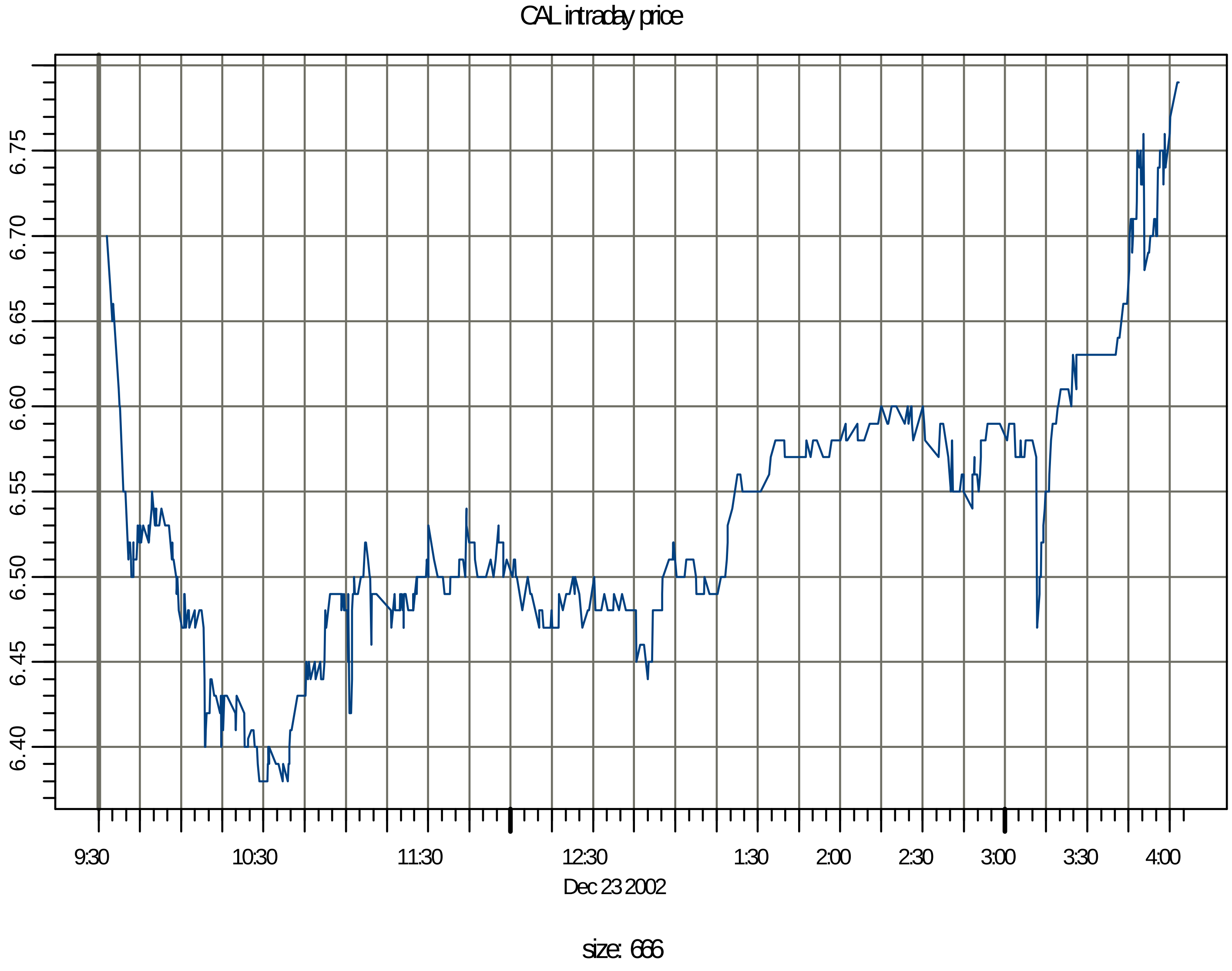
  discretization error v.s. microstructure noise
- ABDL(01): linear interp.; ABDE(01): previous tick interp.

Some authors report a *downward* bias of linear interp. scheme for volatility estimation (e.g., Corsi, et al(01), Barucci-Renò(02)).

Use of previous tick interpolation seems more natural, getting more popularity.



TDW intraday price: 12/23/02



CAL intraday price: 12/23/02

Correlation estimates: CAL-TDW

h (min)	Realized correl prev tick	Realized correl linear	HY
1	0.1516	0.0856	
5	0.1350	0.1028	0.1264
10	0.0909	0.0598	
19.5	0.3574	0.2306	

The "Epps effect"

Correlations of changes in log price*

Interval	AMC Chryster	AMC Ford	AMC GM	Chrysler Ford	Chrysler GM	Ford GM
10 min	.001	.009	-.009	-.014	.007	.055
20 min	.009	.018	.011	.017	.026	.118
40 min	.006	.012	.014	.041	.040	.197
1 hr	-.043	.057	.064	.023	.065	.294
2 hrs	.029	.060	.094	.112	.129	.383
3 hrs	.031	.158	.111	.361	.518	.519
1 day	-.067	.170	.078	.342	.442	.571
2 days	-.020	.223	.186	.336	.449	.572
3 days	-.098	.203	.100	.334	.542	.645

*Reproduction of Table 1 of Epps (79). Data used: prices recorded at 10 min intervals during each of the 125 trading days in the first 6 months of 1971, obtained from records of price and transaction time for each transaction on the NYSE (via previous-tick interpolation).

Empirical fact: *Correlation measurements decrease when sampling frequency increases.*

The "Epps effect"

Reference: T.W. Epps(79), *JASA* 74, 291-298.

Literature

Diffusion estimation from discrete samples:

-MANY(Dacunha-Castelle and Florens-Zmirou (86), Prakasa Rao(88), Yoshida(92), Genon-Catalot and Jacod(93,94), Kesssler(97), ...;

Bibby and SØrensen(95), Pedersen(95), Hansen and Scheinkman(95), Aït-Sahalia(96,02), ...)

Nonsynchronicity has been rarely addressed in the literature on diffusion estimation with discrete samples, at least up to very recently.

PART II. Nonsynchronous Covariation

$(\Omega, \mathcal{F}, (\mathcal{F}_t), P)$

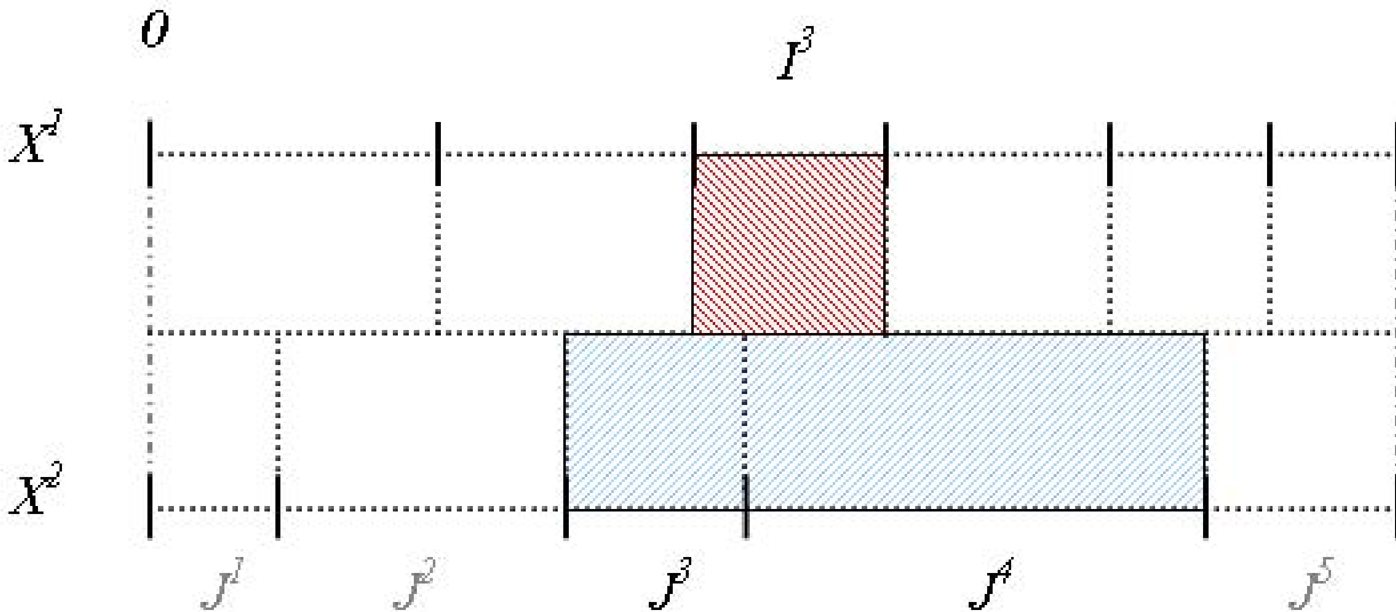
PROCESS:

- X^k $(= X^k(0) + A^k + M^k)$: continuous semimartingales.

SAMPLING TIMES:

- $S^i_{(n)}, T^i_{(n)}$: stopping times, $S^0_{(n)} = T^0_{(n)} = 0$, s.t. $S^i_{(n)} \uparrow \infty$ and $T^i_{(n)} \uparrow \infty$ as $i \to \infty$ a.s. $(n \geq 1)$.
- $I^i_{(n)}(t) := [S^{i-1}_{(n)} \wedge t, S^i_{(n)} \wedge t)$, $J^i_{(n)}(t) := [T^{i-1}_{(n)} \wedge t, T^i_{(n)} \wedge t)$.
- n: index that controls the number of sampling.

Remark: $(I^i_{(n)}(t))$ and $(J^i_{(n)}(t))$ can be mutually dependent.



0
I^3
X^1
X^2
J^1
J^2
J^3
J^4
J^5

For ease of writing...

-$S^i := S^i_{(n)}$, $T^i := T^i_{(n)}$, $I^i(t) := I^i_{(n)}(t)$, $J^i(t) := J^i_{(n)}(t)$.

-$I^i := I^i(T)$, $J^i := J^i(T)$, $T < \infty$ a terminal time of sampling.

STATISTIC:

$$\{X^1, X^2; (I^i), (J^j)\} := \sum_{i,j=1}^{\infty} \Delta X^1(I^i)\Delta X^2(J^j)1_{\{I^i \cap J^j \neq \emptyset\}}, \qquad (4)$$

where $\Delta X^1(I^i) := X^1_{S^i \wedge T} - X^1_{S^{i-1} \wedge T}$, etc.

Remark:

(1) Does not depend on the *artifact* parameter $h(= T/m)$.

(2) Computational load: $\sim$ realized covariance.

Condition (a-0): $r_n := \max_i |I^i| \vee \max_j |J^j| \xrightarrow{P} 0$ as $n \to \infty$.

Theorem 1 (Consistency) Under (a-0), as $n \to \infty$,

$$\{X^1, X^2; (I^i), (J^j)\} \xrightarrow{P} [X^1, X^2]_T .$$

Shown by

- H.-Yoshida(03,05): diffusions (with deterministic σ and ρ) + independent times.
- H.-Kusuoka(04): continuous semimartingales + stopping times.

A generalized construction of realized covariance – or prelimit of quadratic covariation – when the associated sampling schemes are non-synchronous.

"Nonsynchronous Covariation"

An extended result is shown in H.-Kusuoka(04):

$STATISTIC$: with continuous,adapted processes $(f_t)\,, (g_t)$,

$$\left\{X^1, X^2; f, g; (I^i), (J^j)\right\} := \sum_{i,j=1}^{\infty} f_{S^{i-1}} g_{T^{j-1}} \Delta X^1(I^i) \Delta X^2(J^j) 1_{\{I^i \cap J^j \neq \emptyset\}}. \tag{5}$$

Theorem 2 Under (a-0), as $n \to \infty$,

$$\left\{X^1, X^2; f, g; (I^i), (J^j)\right\} \xrightarrow{P} \int_0^T f_t g_t d\left[X^1, X^2\right]_t.$$

ASYMPTOTIC NORMALITY

Under certain regularity condition, $\left\{X^1, X^2; (I^i), (J^j)\right\}$ is asymptotically normally distributed as $n \to \infty$.

Proved by

- H.-Yoshida(04): diffusions (with deterministic σ and ρ) + independent times.
- H.-Yoshida(06): continuous semimartingales + stopping times (*functional convergence*).

Remark: Dalalyan and Yoshida (06) derive a 2nd order asymptotic expansion of the estimator in the setup of H.-Yoshida(04).

Example: diffusion-type processes

PROCESS: Diffusions with *deterministic* $\sigma(t)$.

- X_t^k: log-price processes:

$$dX_t^k = \mu_t^k dt + \sigma_t^k dW_t^k, \; k = 1, 2,$$

with $d\left[W^1, W^2\right]_t = \rho_t dt$.
$\sigma^k(\cdot) \neq 0$ bdd, deterministic; $\mu^k(\cdot)$ prog. meas. function;
$\rho(\cdot) \in (-1, 1)$ deterministic (*unknown*).

Integrated variance/covariance estimation:

$$U_{n,T} := \begin{bmatrix} \left\{X^1, X^2; (I^i), (J^j)\right\} \\ RV\left(X^1; (I^i)\right) \\ RV\left(X^2; (J^j)\right) \end{bmatrix} := \begin{bmatrix} \sum_{i,j} \Delta X^1(I^i)\Delta X^2(J^j)K_{ij} \\ \sum_i \Delta X^1(I^i)^2 \\ \sum_j \Delta X^2(J^j)^2 \end{bmatrix}. \tag{6}$$

$$\theta := \begin{bmatrix} \left[X^1, X^2\right]_T \\ \left[X^1, X^1\right]_T \\ \left[X^2, X^2\right]_T \end{bmatrix} := \begin{bmatrix} \int_0^T \sigma_t^1 \sigma_t^2 \rho_t dt \\ \int_0^T \left(\sigma_t^1\right)^2 dt \\ \int_0^T \left(\sigma_t^2\right)^2 dt \end{bmatrix}. \tag{7}$$

Joint asymptotic normality (H.-Yoshida(05b)).

- The ‘distribution functions’ assoc. with the sampling times:

$$H_n^1(t) := \sum_i |I^i(t)|^2,\ H_n^2(t) := \sum_j |J^j(t)|^2,$$

$$H_n^{1\cap 2}(t) := \sum_{i,j} |(I^i \cap J^j)(t)|^2,$$

$$H_n^{1*2}(t) := \sum_{i,j} |I^i(t)||J^j(t)|K_{ij},$$

cf. $H_n^k(t), k = 1, 2,$ “quadratic variation of time” (Mykland and Zhang 05).

Theorem 3 Under certain regularity conditions, as $n \to \infty$,

$$b_n^{-1/2}\left(U_{n,T} - \theta\right) \xrightarrow{\mathcal{L}} N(0, \mathbb{C}),$$

where $\mathbb{C} := \left(\mathbb{C}^{(l,k)}\right)_{0 \le l,k \le 2}$ comprises

$$\mathbb{C}^{(0,0)} := \int_0^T \left(\sigma_t^1 \sigma_t^2\right)^2 dH^{1*2}(t) + \int_0^T \left(\sigma_t^1 \sigma_t^2 \rho_t\right)^2 d(H^1 + H^2 - H^{1\cap 2})(t),$$

$$\mathbb{C}^{(l,l)} := 2\int_0^T \left(\sigma_t^l\right)^4 dH^l(t),\ l = 1, 2,$$

$$\mathbb{C}^{(l,0)} := \mathbb{C}^{(0,l)} := 2\int_0^T \left(\sigma_t^l\right)^2 \left(\sigma_t^1 \sigma_t^2 \rho_t\right) dH^l(t),\ l = 1, 2,$$

$$\mathbb{C}^{(2,1)} := \mathbb{C}^{(1,2)} := 2\int_0^T \left(\sigma_t^1 \sigma_t^2 \rho_t\right)^2 dH^{1\cap 2}(t).$$

where H^1, H^2, H^{1*2}, $H^{1\cap 2}$ are nontrivial, non-random, nondecreasing, continuous functions on $[0, T]$ that are the respective weak limits of H_n^1, H_n^2, H_n^{1*2}, $H_n^{1\cap 2}$ rescaled by b_n^{-1}.

SOME STATISTICAL PROPERTY

In case of (conditional) Gaussian diffusions with $\mu = 0$, $\{X^1, X^2; (I^i), (J^j)\}$ is

- the MLE for $[X^1, X^2]_T$; Mykland (06).
- the best unbiased estimator among all unbiased estimators for $[X^1, X^2]_T$; Dalalyan and Yoshida (06).

Consider 3 cases with:

- $b_n = n^{-1}$;
- $\big(S^i\big)$, $\big(T^i\big)$ independent of X.

CASE I: Synchronous equidistant sampling: $I^i \equiv J^i$, $|I^i| \equiv \frac{T}{n}$.

$$\mathbb{C} = T \begin{bmatrix} \int_0^T \left(\sigma_t^1 \sigma_t^2\right)^2 (1+\rho_t^2) dt & & \\ 2\int_0^T \left(\sigma_t^1\right)^2 (\sigma_t^1 \sigma_t^2 \rho_t) dt & 2\int_0^T \left(\sigma_t^1\right)^4 dt & \\ 2\int_0^T \left(\sigma_t^2\right)^2 (\sigma_t^1 \sigma_t^2 \rho_t) dt & 2\int_0^T \left(\sigma_t^1 \sigma_t^2 \rho_t\right)^2 dt & 2\int_0^T \left(\sigma_t^2\right)^4 dt \end{bmatrix}. \tag{8}$$

cf. Jacod(94), Barndorff-Nielsen and Shephard(04).

CASE II: Nonsynchronous alternating sampling at odd/even times:

- $\Pi^1 := \left\{ \frac{1}{2n}T, \frac{3}{2n}T, \ldots, \frac{2k-1}{2n}T, \ldots, \right\} \cup \{0, T\}$. ("odd" times)
- $\Pi^2 := \left\{ \frac{2}{2n}T, \frac{4}{2n}T, \ldots, \frac{2k}{2n}T, \ldots \right\} \cup \{0, T\}$. ("even" times)

Then,

$$\mathbb{C} = T \begin{bmatrix} \int_0^T \left(\sigma_t^1 \sigma_t^2\right)^2 (2 + \frac{3}{2}\rho_t^2) dt & & \\ 2\int_0^T \left(\sigma_t^1\right)^2 (\sigma_t^1 \sigma_t^2 \rho_t) dt & 2\int_0^T \left(\sigma_t^1\right)^4 dt & \\ 2\int_0^T \left(\sigma_t^2\right)^2 (\sigma_t^1 \sigma_t^2 \rho_t) dt & \int_0^T \left(\sigma_t^1 \sigma_t^2 \rho_t\right)^2 dt & 2\int_0^T \left(\sigma_t^2\right)^4 dt \end{bmatrix}. \tag{9}$$

CASE III: Independent Poisson sampling:

- $\left(S^i\right)$, $\left(T^i\right)$: Poisson arrival times, $\lambda^k = np^k$, $p^k \in (0,\infty)$, $k = 1,2$.
- $I^i := \left[S^{i-1} \wedge T, S^i \wedge T\right)$ and $J^i := \left[T^{i-1} \wedge T, T^i \wedge T\right)$.

Then, $\mathbb{C} := \left(\mathbb{C}^{(l,k)}\right)_{0 \le l,k \le 2}$, where

$$\mathbb{C}^{(0,0)} = \left(\frac{2}{p^1} + \frac{2}{p^2}\right) \int_0^T \left(\sigma_t^1 \sigma_t^2\right)^2 dt + \left(\frac{2}{p^1} + \frac{2}{p^2} - \frac{2}{p^1 + p^2}\right) \int_0^T \left(\sigma_t^1 \sigma_t^2 \rho_t\right)^2 dt$$

$$\mathbb{C}^{(l,l)} = \frac{4}{p^l} \int_0^T \left(\sigma_t^l\right)^4 dt, \; l = 1,2,$$

$$\mathbb{C}^{(l,0)} = \frac{4}{p^l} \int_0^T \left(\sigma_t^l\right)^2 \left(\sigma_t^1 \sigma_t^2 \rho_t\right) dt, \; l = 1,2,$$

$$\mathbb{C}^{(2,1)} = \frac{4}{p^1 + p^2} \int_0^T \left(\sigma_t^1 \sigma_t^2 \rho_t\right)^2 dt.$$

Correlation estimation:

Assume $\sigma^l(t) \equiv \sigma^l > 0$, $l = 1, 2$, and $\rho(t) \equiv \rho$ (*unknown*).

CORRELATION ESTIMATORS:

$$R_n^{(1)} \equiv \frac{1}{T} \frac{\sum_{i,j} \Delta X^1(I^i) \Delta X^2(J^j) 1_{\{I^i \cap J^j \neq \emptyset\}}}{\sigma^1 \sigma^2} \quad (\sigma^l \text{ known}), \tag{10}$$

$$R_n^{(2)} \equiv \frac{\sum_{i,j} \Delta X^1(I^i) \Delta X^2(J^j) 1_{\{I^i \cap J^j \neq \emptyset\}}}{\sqrt{\sum_i \{\Delta X^1(I^i)\}^2} \sqrt{\sum_j \{\Delta X^2(J^j)\}^2}} \quad (\sigma^l \text{ known/unknown}). \tag{11}$$

They are consistent as $n \to \infty$. Moreover,

- CASE I: Synchronous sampling:

$$\sqrt{n}\left(R_n^{(1)} - \rho\right) \xrightarrow{\mathcal{L}} N\left(0, (1+\rho^2)\right),$$
$$\sqrt{n}\left(R_n^{(2)} - \rho\right) \xrightarrow{\mathcal{L}} N\left(0, (1-\rho^2)^2\right).$$

- CASE II: Nonsynchronous alternating sampling:

$$\sqrt{n}\left(R_n^{(1)} - \rho\right) \xrightarrow{\mathcal{L}} N\left(0, 2(4+3\rho^2)\right),$$
$$\sqrt{n}\left(R_n^{(2)} - \rho\right) \xrightarrow{\mathcal{L}} N\left(0, 2\left\{(1-\rho^2)^2 + (3-\rho^2)\right\}\right).$$

- CASE III: Poisson sampling:

$$\sqrt{n}\left(R_n^{(1)}-\rho\right) \xrightarrow{\mathcal{L}} N\left(0, c_\rho^{(1)}\right),$$
$$\sqrt{n}\left(R_n^{(2)}-\rho\right) \xrightarrow{\mathcal{L}} N\left(0, c_\rho^{(2)}\right),$$

where

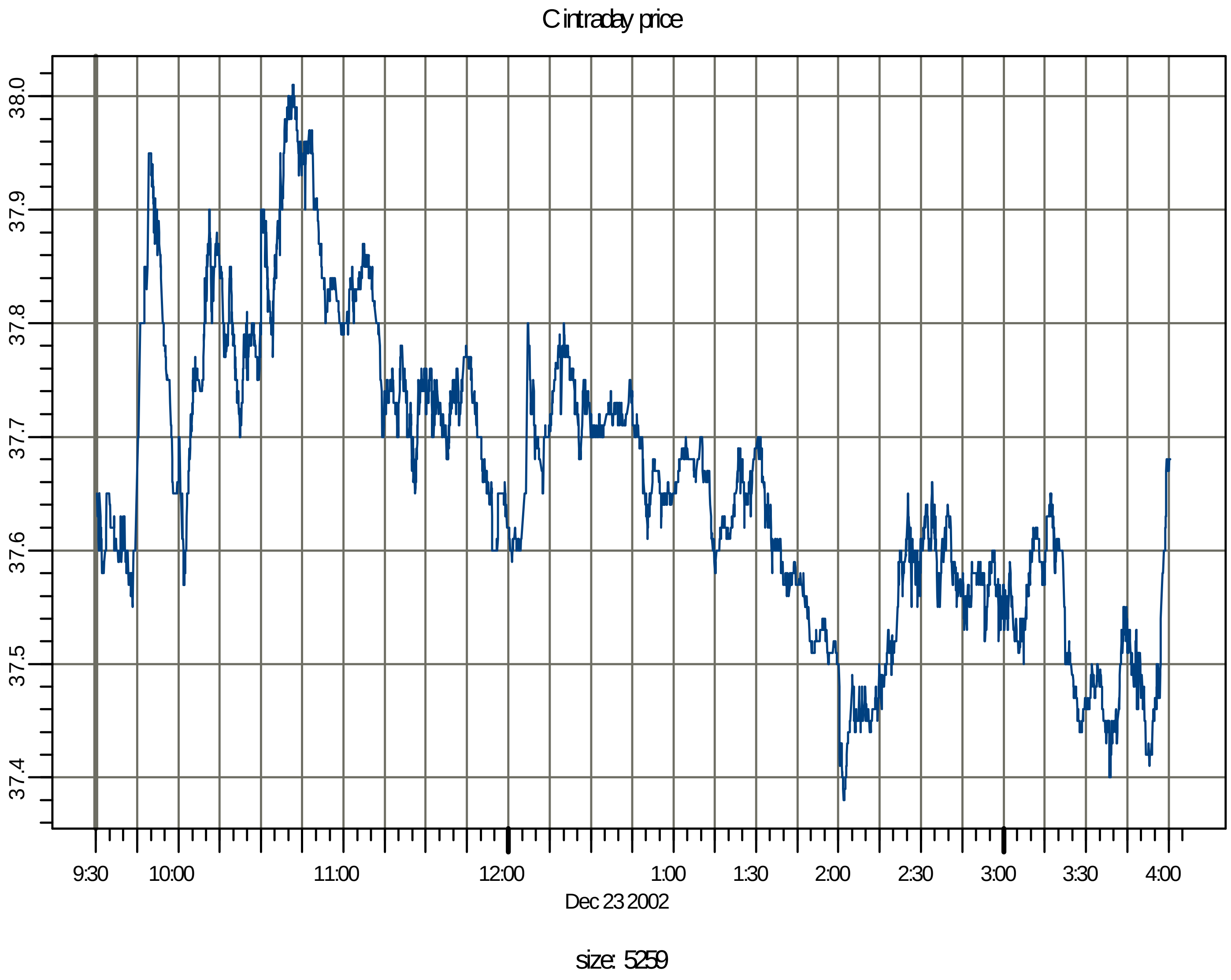
$$c_\rho^{(1)} := \frac{2}{T}\left\{\left(\frac{1}{p^1}+\frac{1}{p^2}\right)+\left(\frac{1}{p^1}+\frac{1}{p^2}-\frac{1}{p^1+p^2}\right)\rho^2\right\}, \tag{12}$$
$$c_\rho^{(2)} := \frac{1}{T}\left\{2\left(\frac{1}{p^1}+\frac{1}{p^2}\right)-\left(\frac{1}{p^1}+\frac{1}{p^2}+\frac{2}{p^1+p^2}\right)\rho^2+\frac{2}{p^1+p^2}\rho^4\right\}. \tag{13}$$

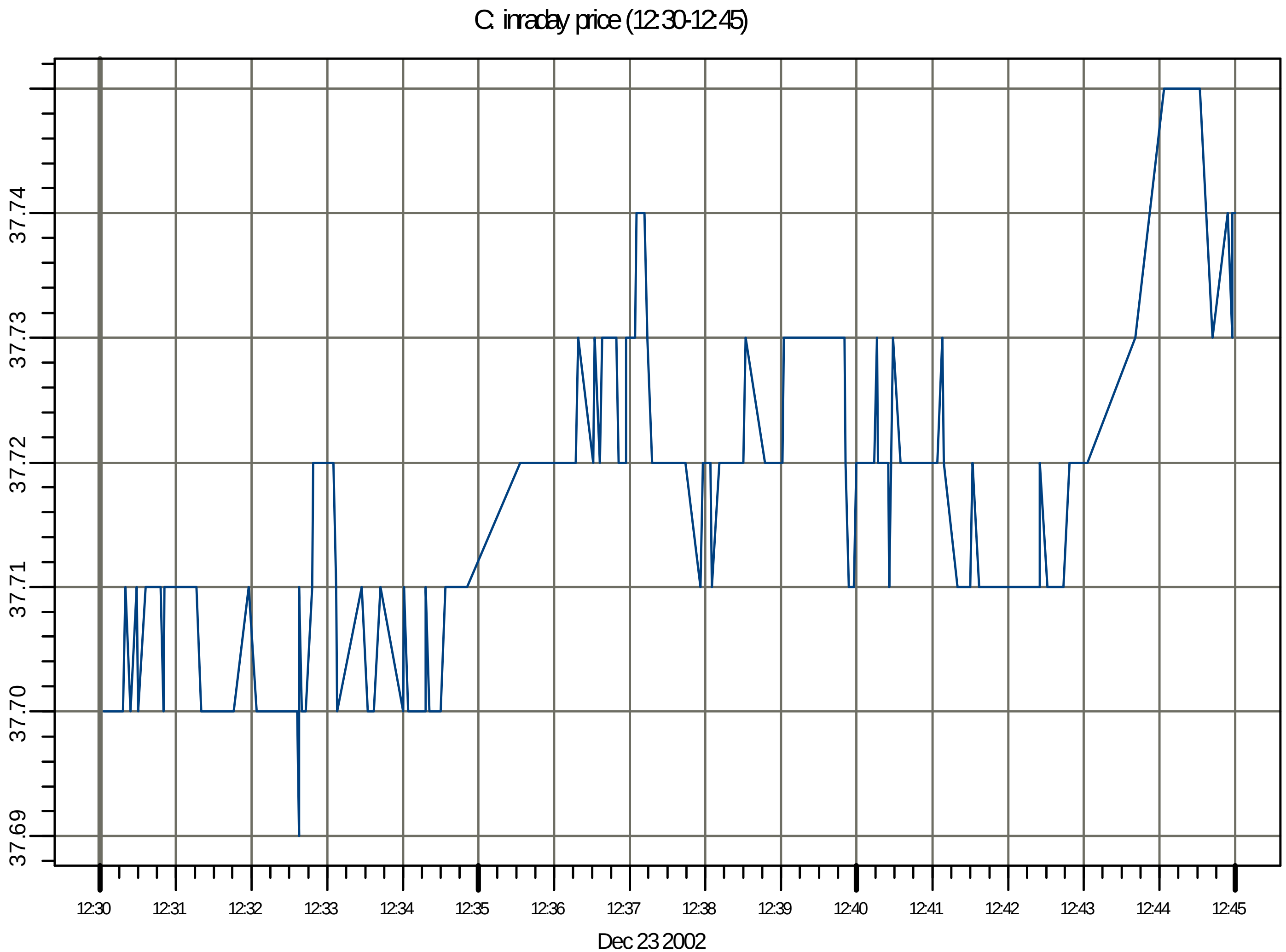
Remark: $c_\rho^{(1)} \geq c_\rho^{(2)}$ ($c_\rho^{(1)} = c_\rho^{(2)} \iff \rho = 0$).

PART III. Getting Closer to Reality - Next Step

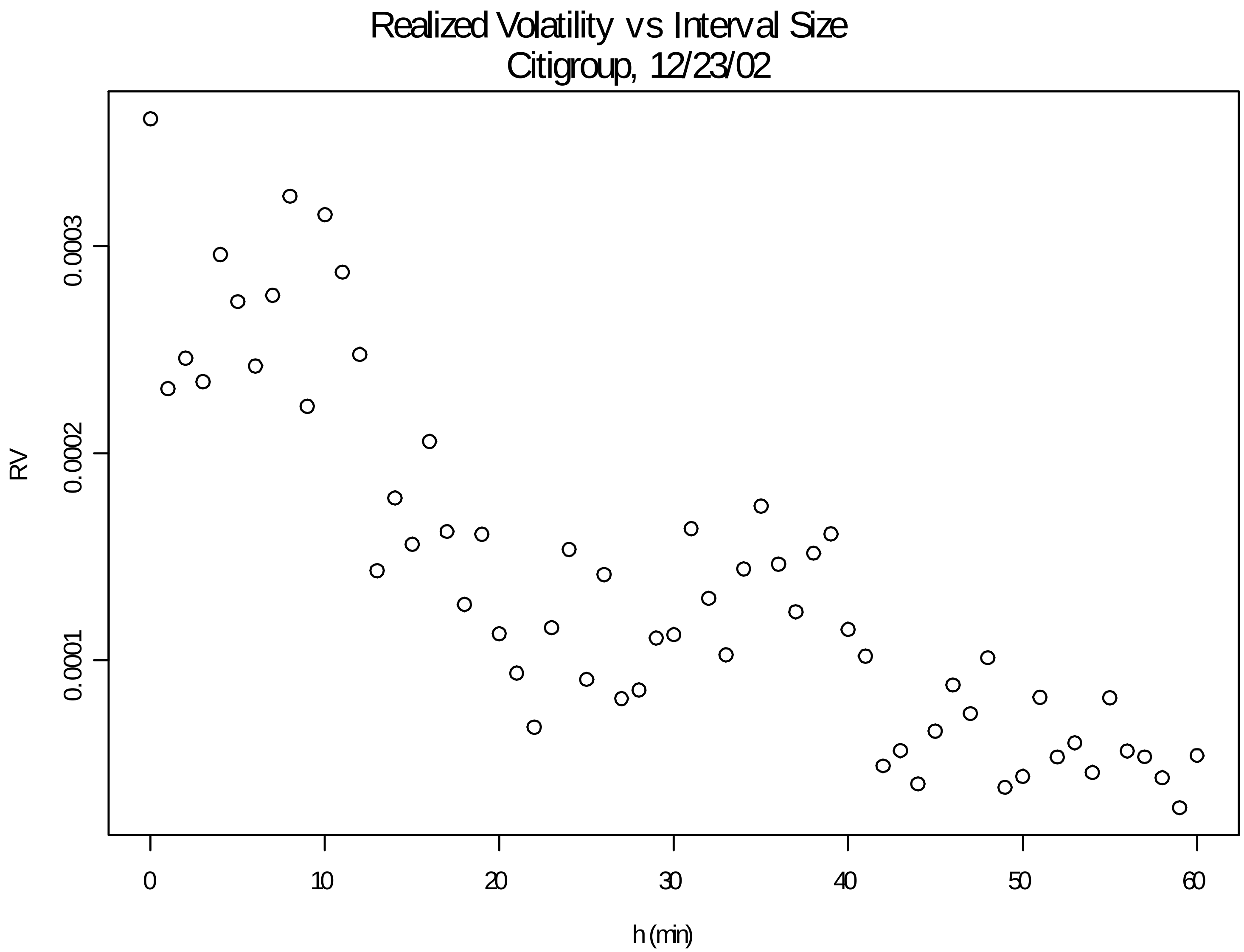
Case with *measurement error* - *microstructure noise contamination*

Look at the intraday price of Citigroup (**C**) on 12/23/02.


C intraday price
38.0
37.9
37.8
37.7
37.6
37.5
37.4
9:30
10:00
11:00
12:00
1:00
1:30
2:00
2:30
3:00
3:30
4:00
Dec 23 2002
size: 5259



C: inraday price (12:30-12:45)
37.69
37.70
37.71
37.72
37.73
37.74
12:30
12:31
12:32
12:33
12:34
12:35
12:36
12:37
12:38
12:39
12:40
12:41
12:42
12:43
12:44
12:45
Dec 23 2002


Realized Volatility vs Interval Size
Citigroup, 12/23/02
RV
0.0001
0.0002
0.0003
0
10
20
30
40
50
60
h (min)

A reasonable model: for latent log-price ("efficient price") process X (semimartingale), the actual observed log-prices $\{Y_{t_i}, i = 0, ..., m\}$ are generated as

$$Y_{t_i} = X_{t_i} + \epsilon_{t_i} \tag{14}$$

where ϵ_{t_i} is a noise term, independent of X (e.g., Aït-Sahalia, Mykland, Zhang, 2005, RFS).

(*Note*: the modeling does not require that ϵ_t exists for every t, but only at every sampling time t_i.)

"**hidden semimartingale model**"

How to compromise; math. finance v.s. statistics?

Each ϵ_{t_i} is too small to be exploited for an arbitrage; however, through propagation they can hinder the estimation of integrated variance.

In fact, if $\epsilon_{t_i} \sim IID(0, v^2)$, e.g., then

$$RV(Y) \equiv \sum_{i=1}^{m} \left(Y_{t_i} - Y_{t_{i-1}}\right)^2 = 2m \times v^2 + O_p\left(m^{1/2}\right), \tag{15}$$

i.e., the naive realized volatility *not* estimates the true integrated variance *but the (rescaled) variance of the noise* (Zhang, Mykland, and Aït-Sahalia, JASA, 2005) !

Toward covariance/correlation estimation

- Microstructure noise contamination can also affect integrated covariance/correlation estimation.
  - It is possible to give an explanation for the Epps effect in the hidden semimartingale modelling.
- Dealing with nonsynchronicity and microstructure noise contamination is a frontier of the current research activities in the field.
  - Zhang (06) discusses the Epps effect via the realized correlation in the presence of nonsynchronicity and microstructure noise.
  - Griffin and Oomen (2006) report a severe downward bias of the nonsynchronous covariation in their empirical analysis using five stocks traded in NYSE.
  - Voev and Lunde (2006) propose a bias correction version of the nonsynchronous covariation under a general noise specification.

CONCLUSION

- "Integrated covariance" estimation for nonsynchronously sampled continuous semimartingales.
- The proposed procedure (*nonsynchronous covariation*) does not depend on any "synchronization" of original data.
- The nonsynchronous covariation possesses: *consistency* and *asymptotic normality* as $n \to \infty$. Also have some good properties.
- *Nonsynchronicity* + *microstructure noise*, work to be done.

REFERENCES

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