News from sequential analysis for stochastic delay differential equation

Uwe Küchler [1] and Vyacheslav Vasiliev [2]

Abstract: Let $X = (X, (t), t \geq 0)$ satisfy the equation

$$dX(t) = (aX(t) + bX(t-1))dt + dW(t),\ t \geq 0,$$

where $\vartheta = (a, b)^T$ is unknown. Based on the observation of X we construct for all $\varepsilon > 0$ and $q \geq 2$ a sequential plan $(T(\varepsilon), \vartheta(\varepsilon))$ with

$$\mathrm{E}_\vartheta \parallel \vartheta(\varepsilon) - \vartheta \parallel_q^2 \leq \varepsilon \quad \text{and} \quad \lim_{\varepsilon \to 0} \vartheta(\varepsilon) = \vartheta\ P_\vartheta - a.s.,\ \vartheta \in \Theta.$$

Moreover we determine the convergence rate of the estimator , i.e. we find the rates of increase of the observation time $T(\varepsilon)$ for $\varepsilon \downarrow 0$ and for every ϑ from different subsets of Θ. Here Θ equals R_2 with the exception of two curves.

The construction of $(T(\varepsilon), \vartheta(\varepsilon))$ is carried out step by step and by combining different procedures corresponding to different subsets of Θ. The presented method is applicable to more general delay differential equations and other linear regression models.

[1] Institute of Mathematics / Humboldt University Berlin
[2] Department of Applied Mathematics and Cybernetics / Tomsk State University