CONTINUOUS TIME INTERPRETATION OF DISCRETE-TIME MARKET DATA REVISITED

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ABSTRACT

It is a widely held belief throughout the financial literature that daily and weekly returns from stocks must be uncorrelated and that any deviation from this principle entails some sort of "inefficiency," or even an opportunity for a "free-lunch" of some sort, which, generally, capital markets cannot tolerate for very long. The origin of this assumption seems to be P. Samuelson's work [2]. Furthermore, in one of the most comprehensive and widely referenced statistical studies of historic market data, going back for nearly 100 years and presented in [1], Campbell, Lo and MacKinlay conclude that: *"The average variance ratio with* $q = 2$ *is* 0.96 *for the* 411 *individual securities, implying that there is negative serial correlation on average. For all stocks, the average serial correlation is* -4%*, and* -5% *for the smallest 100 stocks. However, the serial correlation is both statistically and economically insignificant and provides little evidence against the random walk hypothesis. For example, the largest average* $\psi^*(q)$ *statistics over all stocks occurs for* $q = 4$ *and is* -0.90 *(with a cross sectional standard deviation of 1.19); the largest average* $\psi^*(q)$ *for the 100 smallest stocks is* -1.67 *(for* $q = 2$ *with a cross sectional standard deviation of* 1.75*). These results are consistent with French and Roll's (1986) finding that daily returns of individual securities are slightly negatively autocorrelated."* Influenced by these results, most financial models assume that stock prices follow geometric Brownian motion (and any significant departure from such models is viewed as "heresy"). However, we will show that the serial correlation found by Campbell, Lo and MacKinlay is just as consistent with discrete-time samples of the same frequency that come from diffusion processes that are profoundly different from geometric Brownian motion. We will show that, typically, i.e., for most diffusion models, the serial correlation converges to 0 as the sampling frequency converges to 0 and that, therefore, depending on the frequency of the sampling, an auto-correlation that appears to be "small," or only "slightly negative," might actually be very significant. We will discuss the limitations of the $\psi^*(q)$ statistics used in [1] and will show that price models that allow for certain self-regulatory behavior are not inconsistent with the underlying principles of financial economics and, at the same time, are entirely consistent with most statistical studies of market data.

[1] J. Campbell, A. Lo and A. MacKinlay (1997). The Econometrics of Financial Markets. Princeton University Press.

[2] Paul Samuelson (1965). Proof that Properly Anticipated Stock Prices Fluctuate Randomly. Industrial Management Review, 6, pp. 41–49.