Estimation of Spectral Functionals for Continuous-time Stationary Models

Mamikon Ginovyan, Boston University

Abstract

Suppose we observe a finite realization $\{X(t),\, 0 \leq t \leq T\}$ of a zero mean real-valued continuous-time stationary Gaussian process $X(t)$ with an *unknown* spectral density function $\theta(\lambda)$, $\lambda \in R^1$. We assume that $\theta(\lambda)$ belongs to a given class $\mathbf{\Theta} \subset L^p(R^1)$ $(p > 1)$ of spectral densities possessing some smoothness properties. Let $\Phi(\cdot)$ be some *known* functional, the domain of definition of which contains $\mathbf{\Theta}$.

The problem is to estimate the value $\Phi(\theta)$ of the functional $\Phi(\cdot)$ at an unknown point $\theta \in \mathbf{\Theta}$, and to investigate the asymptotic (as $T \to \infty$) properties of the suggested estimators. The main objective is construction of asymptotically efficient estimators for $\Phi(\theta)$.

We show that the statistic $\Phi(I_T)$, where $I_T = I_T(\lambda)$ is the periodogram of the underlying process $X(t)$, is an asymptotically efficient estimator for a linear functional $\Phi(\theta)$, while for a nonlinear smooth functional $\Phi(\theta)$ an asymptotically efficient estimator is $\Phi(\widehat{\theta}_T)$, where $\widehat{\theta}_T$ is a suitable sequence of $T^{1/2}$-consistent estimators of the unknown spectral density $\theta(\lambda)$.