Approximations and limit theorems for log-likelihood ratio processes

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(joint work with E. Valkeila)

Abstract

We consider a sequence

$$\mathbb{E}^n = (\Omega^n, \mathcal{F}^n, \mathbb{F}^n = (\mathcal{F}^n_t)_{t\geq 0}, (\mathsf{P}^{n,\vartheta})_{\vartheta\in\Theta})$$

of filtered statistical models, where Θ is an open subset of $\mathbb{R}^k$. Let $\varphi_n \to 0$ be a normalizing sequence of positive definite $k \times k$ matrices and let $Z^{n,\vartheta}_t$ be the density process of $\mathsf{P}^{n,\vartheta_0+\varphi_n\vartheta}$ with respect to $\mathsf{P}^n := \mathsf{P}^{n,\vartheta_0}$. In the talk we discuss conditions under which there is a quadratic approximation of the log-likelihood ratio processes $\log Z^{n,\vartheta}$ in the following sense:

$$\sup_{t\geq 0}\Big|\log Z^{n,\vartheta_n}_t - \Big(\vartheta_n^\top w^n_t - \frac{1}{2}\vartheta_n^\top\langle w^n, w^n\rangle_t\vartheta_n\Big)\Big| \xrightarrow{\mathsf{P}^n} 0, \quad n\to\infty,$$

for each bounded sequence $\{\vartheta_n\}$ in $\mathbb{R}^k$, where $w^n_t = (w^{n,1}_t, \dots, w^{n,k}_t)$, $w^n_0 = 0$, is a locally square-integrable martingale on $(\Omega^n, \mathcal{F}^n, \mathbb{F}^n, \mathsf{P}^n)$ with values in $\mathbb{R}^k$ for each n, and $\langle w^n, w^n\rangle_t$ is its quadratic characteristic. Having proved such an approximation, it is easy to deduce general conditions for local asymptotic normality (LAN) or local asymptotic mixed normality (LAMN) at ϑ_0. The most intriguing results are three theorems showing necessity of conditions under which we prove the approximation result. They provide a new information even in the case of i.i.d. observations.