On the Invariant Measure for Branching Diffusions with Immigration

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Abstract

In this talk we consider a system of finitely many particles living in $\mathbb{R}^d$ and moving independently of each other on paths given by the solution to an SDE

$$d\xi_t = b(\xi_t)dt + \sigma(\xi_t)dW_t.$$

Each particle branches (i.e. dies and produces a random number of offspring) at a random time according to a position-dependent branching rate κ. A particle dying at position $x \in \mathbb{R}^d$ gives rise to $k \in \mathbb{N}_0$ offspring particles with probability $p_k(x)$, the offspring particles selecting their position according to $\bigotimes_{j=1}^k Q(x, \cdot)$ if $k \geq 1$, where $Q(x, \cdot)$ is a probability kernel on $\mathbb{R}^d$. In addition, there is immigration occuring at a constant rate $c > 0$, adding one new particle to the pre-existing configuration in a position selected according to a probability law π on $\mathbb{R}^d$. The resulting process of finite particle configurations is called a *branching diffusions with immigration* (BDI). Under a suitable condition of subcritical reproduction, a BDI is positive Harris recurrent with invariant probability measure m on the configuration space and finite associated intensity measure $\bar{m}$ on the one-particle space $\mathbb{R}^d$.

In recent years, the problem of statistical inference for this class of processes has attained growing interest. Statistical problems for ergodic BDIs that have been considered in the literature include: local asymptotic normality for parametric models where b, κ, p_k, Q, c and π depend on a finite-dimensional parameter; non-parametric estimation of the branching rate κ from continuous-time obervations; and non-parametric estimation of the diffusion coefficient σ from discrete-time observations. In the study of these problems, a crucial role is being played by the properties of m and $\bar{m}$. We investigate some of these properties. For example, we discuss the validity of certain integrability resp. moment conditions and give conditions ensuring the existence of a continuous and bounded density.