Estimating discontinuous periodic signals in a time inhomogeneous diffusion

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(joint work with Y. Kutoyants)

Abstract

Let Q^0 denote the law of a diffusion $\xi = (\xi_t)_{t\geq 0}$ having some known T-periodic time dependent contribution $t \to \lambda(t)$ in the drift. Let Q^ϑ denote the law of the process if on periodically occurring intervals of known length a

$$[kT + \vartheta, kT + \vartheta + a) \quad , \quad k \in \mathbb{N}_0$$

a T-periodic signal $t \to \lambda^*(t)$ is added to the drift. We wish to estimate the parameter ϑ from observation of a trajectory of ξ over a long time interval $[0, nT]$ as $n \to \infty$. We deal with convergence of likelihoods in local models at ϑ, with local scale $\frac{1}{n}$ and local parameter h, and with convergence of maximum likelihood (MLE) and Bayes estimators (BE). At every stage n of the asymptotics, the parametrization is Hölder continuous – in the sense of Hellinger distance – of order $\frac{1}{2}$. The limit of local models has likelihoods of type $h \to e^{W_h - \frac{1}{2}|h|}$, $h \in \mathbb{R}$, with double-sided Brownian motion W. Properties of certain estimators in the limit model have been studied since Ibragimov and Khasminskii (1981).

On the probabilistic side, assuming positive Harris recurrence of $(\xi_{kT})_{k\in\mathbb{N}_0}$ and exploiting the periodicity structure, we prove a Harris property for the chain of T–segments

$$\left(\left(\xi_{(k-1)T+r} \right)_{0\leq r\leq T} \right)_k \quad , \quad k \in \mathbb{N}$$

which allows to formulate limit theorems for certain martingales and certain functionals of the process $\xi = (\xi_t)_{t\geq 0}$ which is non-homogeneous in time. This provides approximations to the log-likelihood ratios in local models at ϑ and ist the key to local asymptotics.

For the limit model, it is known that a BE is better than MLE. On the statistical side, our approach relys on contiguity and makes extensive use of LeCam's '3rd lemma', in contrast to work done so far: we prove a local asymptotic equivariance property of a BE sequence, and a local asymptotic minimax bound under quadratic loss which is attained by this BE sequence.