Pseudo-divergence test statistics for multidimensional diffusion processes observed at discrete times

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Abstract

We propose the use of pseudo ϕ-divergences as statistics in the problem of hypotheses testing for a multidimensional parametric diffusion process

$$\mathrm{d}X_t = b(X_t, \theta_1)\mathrm{d}t + \sigma(X_t, \theta_2)\mathrm{d}W_t,$$

from discrete observations $\{X_{t_i}, i = 0, \ldots, n\}$ with $t_i = i\Delta_n$, $i = 0, 1, \ldots, n$, under the asymptotic scheme $\Delta_n \to 0$, $n\Delta_n \to \infty$ and $n\Delta_n^2 \to 0$. The class of ϕ-divergences is wide and includes several special members like Kullback-Leibler, Rényi, power and α-divergences. We derive the asymptotic distribution of the test statistics based on the estimated pseudo ϕ-divergences. The limiting law takes different forms depending on the regularity of ϕ. These convergence results differ from the classical results for independent and identically distributed random variables. Furthermore, we present some analytic approximations of the power function of these statistics. Numerical analysis is used to show the small sample properties of the proposed test in terms of empirical level and power.