Statistical problems related to excitation threshold and reset value of membrane potentials

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Abstract

A commonly used model for spiking neurons is to assume that between spikes the membrane potential is given by a diffusion process $(X_t)_{t\geq 0}$ which is a solution of an SDE

$$\mathrm{d}X_t = b(X_t)\mathrm{d}t + \sigma(X_t)\mathrm{d}B_t$$

where $(B_t)_{t\geq 0}$ is a standard Brownian motion. The spiking behavior is usually explained as follows. Whenever a certain *excitation threshold* S is reached, a spike occurs. Thereafter the potential is set down to a certain *reset value* x_0 again.

In applications it is sometimes possible to observe the diffusion process $(X_t)_{t\geq 0}$ between spikes by estimating the drift coefficient $b(\cdot)$ and the diffusion coefficient $\sigma(\cdot)$ from real data. Nevertheless, one has to determine S and x_0 to fix the model. However, in real data this is not obvious at all.

One possible issue is to view S and x_0 as parameters in a statistical model and to estimate these. We discuss four cases where we assume the diffusion process X between spikes is given by a *Brownian motion with drift*, a *geometric Brownian motion*, an *Ornstein-Uhlenbeck* process or a *Cox-Ingersoll-Ross* process. We assume further that we observe iid spike times interpreted as level crossing times of X from x_0 to S. The first two cases are very similar and one can compute explicitly the *maximum likelihood* estimator. Moreover, we use LAN theory to get optimal results. The cases OU and CIR process are treated by a *minimum distance* approach based on the comparison of empirical and true Laplace transform in a Hilbert space norm. It will be shown that all estimators are strongly consistent and asymptotically normal. Further, applications to real and simulated data are given.