Statistical analysis of reliability of redundant systems with "warm" stand-by units

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Abstract

Let us consider redundant systems with one main unit and $m-1$ stand-by units operating in "warm" conditions, i.e. under lower stress than the main one. We shall use notation $S(1,m-1)$ for such systems. The problem is to obtain a confidence interval for the cumulative distribution function of redundant systems using failure data of two groups of units, the first group functioning in "hot" and second — in "warm" conditions. We suppose that switching from "warm" to "hot" conditions does not do any damage to units. Mathematical formulation of fluent switching from "warm" to "hot" conditions of standby units is given using the well known Sedyakin's and accelerated failure time (AFT) models. Denote by $T_1,\ F_1$ and f_1 the failure time, the c.d.f. and the probability density function of the main unit. The failure times of the stand-by units denote by $T_2,\dots,T_m$. In "hot" conditions their distribution functions are also F_1. In "warm" conditions the c.d.f. of T_i is F_2 and the p.d.f is $f_2,\ i=2,\dots,m$. If a stand-by unit is switched to "hot" conditions, its c.d.f. is different from F_1 and F_2. The failure time of the system $S(1,m-1)$ is $T^{(m)}=T_1\vee T_2\vee\cdots\vee T_m$. Denote by K_j and k_j the c.d.f. and the p.d.f. of $T^{(j)}$, respectively, $(j=2,\dots,m)$, $K_1=F_1,\ k_1=f_1$. The c.d.f. K_j can be written in terms of the c.d.f. K_{j-1} and F_1:

$$K_j(t)=\mathbf{P}(T^{(j)}\le t)=\int_0^t \mathbf{P}(T_j\le t|T^{(j-1)}=y)\,dK_{j-1}(y).$$

The "fluent switch on" hypothesis H_0 formulated in states that

$$f_{T_j|T^{(j-1)}=y}(t)=\begin{cases} f_2(t) & \text{if } t\le y,\\ f_1(t+g(y)-y) & \text{if } t>y;\end{cases};\quad g(y)=F_1^{-1}(F_2(y)).$$

The model implies that

$$K_j(t)=\int_0^t F_1(t+g(y)-y)\,dK_{j-1}(y). \qquad (\star)$$

So the distribution function K_m of the system with $m-1$ stand-by units is defined recurrently using the formula $(\star)$. In particular, if we

suppose that the distribution of units functioning in "warm" and "hot" conditions differ only in scale, i.e. $F_2(t) = F_1(rt)$ for all $t \geq 0$ and some $r > 0$, then $g(y) = ry$. Semi-parametric estimators of cumulative distribution function and mean failure time of redundant system with several stand-by units are proposed. Goodness-of-fit tests for two given models are given.