Different approaches to statistical estimation of the response function

Vladimir Zaiats, Escola Politècnica Superior, Universitat de Vic

Abstract

A general linear model of a time-invariant system with additive stochastic disturbances can be written as follows:

$$y(t) = \sum_{k=1}^{\infty} h_k x(t-k) + \xi(t), \qquad t = 1, 2, \dots \tag{1}$$

in a discrete-time setting, and

$$y(t) = \int_0^{\infty} h(u) x(t-u) du + \xi(t), \qquad t \geq 0, \tag{2}$$

in a continuous-time setting. Here, x and y are the input and output, respectively, at time t and ξ is supposed to be a stationary zero-mean process. Models (1) and (2) are also called SISO, meaning single-input single-output.

In model (1), the complex-valued function

$$H(e^{i\omega}) = \sum_{k=1}^{\infty} h_k e^{-ik\omega}, \qquad -\pi \leq \omega \leq \pi, \tag{3}$$

is the so-called the transfer function. A continuous-time counterpart is

$$H(\lambda) = \int_{-\infty}^{\infty} h(u) e^{-i\lambda u}, \qquad \lambda \in \mathbb{R}. \tag{4}$$

The transfer function plays a very important role for understanding the properties of systems (1) and (2).

Many different methods have been developed for estimation of the transfer function, based either on frequency-domain methods or on time-domain methods. The latter are based on finite parametric models. In practice, it is not reasonable to assume that the "true" model is finitely parameterizable. In this context, the purpose of system modeling is to obtain a model involving a finite number of unknown parameters that provides a "reasonable" approximation to the observed data, rather than to estimate parameters of the "true" system.

In Ljung and Yuan [8] and Ljung and Wahlberg [9], asymptotic properties of the least squares estimator for an approximation of the finite impulse response have been studied. In is shown that the dimension of the model should increase with the

number of data in order to ensure convergence. The book by Hannan and Deistler [4] is a good reference on model selection methods. Goldenshluger [2] applied nonparametric minimax approach to the problem of estimation of the transfer function in model (1).

We discuss the existing methods and extend the existing results to continuous-time models (2). Our interest is also focused at the possibility of application of irregular sampling which has proved to be efficient in estimation of correlation functions and spectral densities, see Masry and co-authors [6, 7, 10, 11, 12, 13, 14]. Other useful references are [1, 3, 5, 15].

References

[1] Elogne, S. N., Perrin, O., Thomas-Agnan, C. (2007) Nonparametric estimation of smooth stationary covariance functions by interpolation methods, *Stat. Inference Stoch. Process.*, to appear.

[2] Goldenshluger, A. (1998) Nonparametric estimation of transfer functions: rates of convergence and adaptation. *IEEE Trans. Inform. Theory*, **44**, no. 2, 644–658.

[3] Hall, P., Fischer, N., Hoffmann, B. (1994) On the nonparametric estimation of covariance functions, *Ann. Statist.*, **22**, no. 4, 2115–2134.

[4] Hannan, E., and Deistler, M. (1988) *The Statistical Theory of Linear Systems*, New York: Wiley.

[5] Jacod, Jean (1993) Random sampling in estimation problems for continuous Gaussian processes with independent increments, *Stochastic Process. Appl.*, **44**, pp. 181–204.

[6] Lii, K.-S., and Masry, E. (1992) Model fitting for continuous time stationary processes from discrete-time data, *J. Multivar. Anal.*, pp. 56–79.

[7] Lii, K.-S., and Masry, E. (1994) Spectral estimation of continuous-time stationary processes from random sampling, *Stochastic Process. Appl.*, **52**, pp. 39–64.

[8] Ljung, Lennart, and Yuan, Zhen-Dong. (1985) Asymptotic properties of black-box identification of transfer functions, *IEEE Trans. Autom. Control*, **AC-30**, no. 6, pp. 514–530.

[9] Ljung, Lennart, and Wahlberg, Bo. (1992) Asymptotic properties of the least squares method for estimating transfer functions and disturbance spectra, *Adv. Appl. Probab.*, **24**, pp. 412–440.

[10] Masry, E. (1978) Poisson sampling and spectral estimation of continuous-time processes, *IEEE Trans. Inform. Theory*, **IT-24**, pp. 173–183.

[11] Masry, E. (1978) Alias-free sampling: an alternative conceptualization and its aplications, *IEEE Trans. Inform. Theory*, **IT-24**, pp. 317–324.

[12] Masry, E. (1980) Discrete-time spectral estimation of continuous-time processes — the orthogonal series method, *Ann. Statist.*, pp. 1100–1109.

[13] Masry, E. (1983) Non-parametric covariance estimation from irregularly spaced data, *Adv. Appl. Probab*, **15**, pp. 113–132.

[14] Masry, E. (1984) Spectral and probability density estimation from irregularly observed data, Time Series Analysis of Irregularly Observed Data, Lect. Notes Statist., vol. 25, pp. 224–250.

[15] Rachdi, M. (2004) Strong consistency with rates of spectral estimation of continuous-time processes: from periodic and poisson sampling schemes, *J. Nonparametr. Stat.*, vol. 16, pp. 349–364.