

Exact asymptotic bias for estimators of the Ornstein-Uhlenbeck process

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Abstract

We study the bias and the bias derivative for a family \mathcal{F} of asymptotically efficient estimators of the Ornstein-Uhlenbeck process. That family contains the maximum likelihood, the conditional maximum likelihood and the empirical estimators.

We show that, if $g(\theta_T)$ is an estimator of $g(\theta)$, where θ is the parameter and $\theta_T \in \mathcal{F}$, then, under mild conditions,

$$T E [g(\theta_T) - g(\theta)] \xrightarrow{T \rightarrow \infty} c_\theta g'(\theta) + \theta g''(\theta),$$

where c_θ is an explicit constant that only depends on the choice of θ_T .

In particular, if θ_T is one of the three previous estimators, one has

$$T E_\theta(\theta_T - \theta) \xrightarrow{T \rightarrow \infty} 2.$$

We also take a glance to the discrete case.