Poisson intensity registration by goodness-of-fit testing

with Olivier Collier

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- **Particularity :** the null hypothesis is composite and nonparametric.

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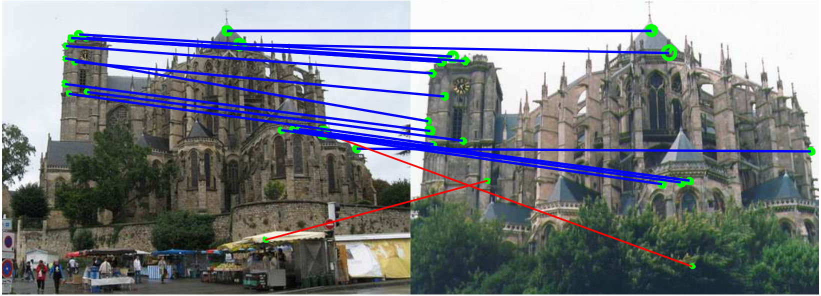

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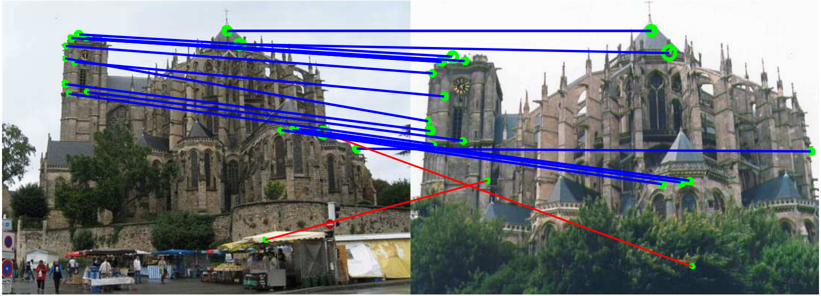

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- If in a ring, two images coincide up to a rotation, then the corresponding Poisson processes have intensities that are equal up to a shift.
- Applying rigorous statistical approach may considerably reduce the number of mismatches.

Reduction to the regression model

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- It is clear that $nX_n(A) \sim \mathcal{P}\big(n \int_A S(x)\, dx\big)$ and therefore

$$X_n(A) \overset{\mathcal{D}}{\approx} \int_A S(x)\, dx + \frac{1}{\sqrt{n}} \mathcal{N}\Big(0, \int_A S(x)\, dx\Big).$$

  or, equivalently,

$$X_n(dx) \overset{\mathcal{D}}{\approx} S(x)\, dx + \frac{\sqrt{S(x)}}{\sqrt{n}} dB(x).$$

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From now on, we deal with the Gaussian white noise model in which :

- we observe $\mathbf{Y}^{\bullet,\#} = \{Y^{\bullet,\#}(x) = (Y(x), Y^{\#}(x)) : x \in [0,1]\}$ s. t.

$$dY^{\bullet,\#}(x) = \begin{bmatrix} f(x) \\ f^{\#}(x) \end{bmatrix} + \sigma d\boldsymbol{W}(x), \qquad \forall x \in [0,1]$$

  where $\boldsymbol{W}$ is a 2D Brownian motion, f and $f^{\#}$ are two unknown 1-periodic signals. (We can abandon the assumption of positiveness.)

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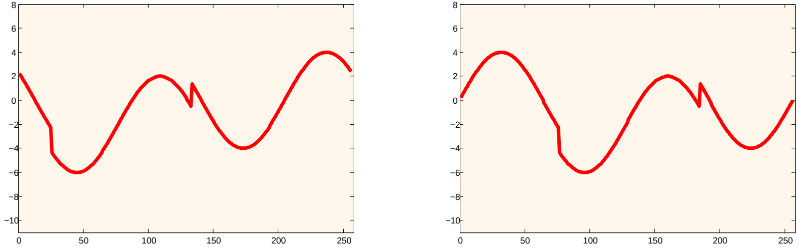
- We wish to test

$$H_0 : \exists\, (b^*, \tau^*) \text{ s.t. } \boxed{f(x) = f^{\#}(x+\tau^*) + b^*}, \ \forall x \in [0,1].$$

An illustration

Two periodic curves coinciding up to a time-shift



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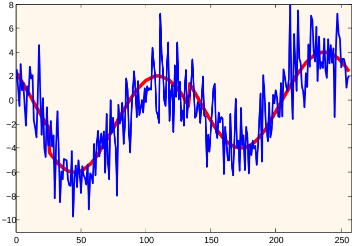
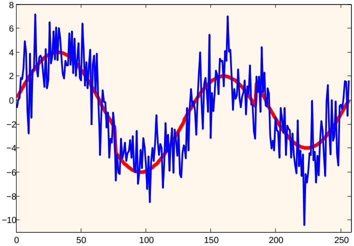
Two periodic curves coinciding up to a time-shift and corrupted by noise





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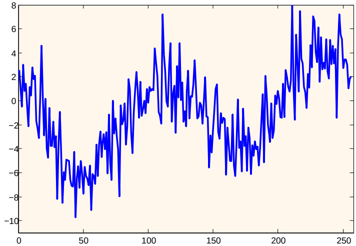
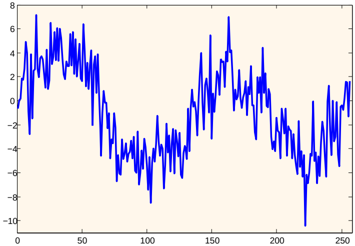
Having observed the noisy curves





is it possible to detect that the original noiseless curves were equal up to a time shift ?

Projection onto the Fourier basis

Gaussian sequence model

- We transform the data into :
$$Y_j = c_j + \sigma\epsilon_j, \qquad Y_j^\# = c_j^\# + \sigma\epsilon_j^\#, \qquad j = 0, 1, 2, \ldots,$$
where $c_j = \int_0^1 f(x)\, e^{2\mathbf{i}\, j\pi x}\, dx$ and $c_j^\# = \int_0^1 f^\#(x)\, e^{2\mathbf{i}\, j\pi x}\, dx$ are the complex Fourier coefficients.

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- We are interested in testing the hypothesis H_0, which translates in the Fourier domain to
$$H_0 : \quad \exists\, \bar{\tau}^* \in [0, 2\pi[\quad \text{s.t.} \quad c_j = e^{-\mathbf{i}\, j\bar{\tau}^*} c_j^\# \quad \forall j = 1, 2, \ldots.$$
- The unknown parameters are assumed to belong to the functional class :
$$\mathcal{F}_{s,L} = \Big\{ \boldsymbol{u} = (u_1, u_2, \ldots) : \sum_{j=1}^{+\infty} j^{2s} |u_j|^2 \leq L^2 \Big\},$$
where the positive real numbers s and L stand for the smoothness and the radius of the class $\mathcal{F}_{s,L}$.

Penalized likelihood ratio (PLR) test

✓ log-likelihood of $\boldsymbol{u}^{\bullet,\#} = (\boldsymbol{u}, \boldsymbol{u}^{\#})$ given $\mathbf{Y}^{\bullet,\#} = (\mathbf{Y}, \mathbf{Y}^{\#})$ is

$$\ell(\mathbf{Y}^{\bullet,\#}, \boldsymbol{u}^{\bullet,\#}) = \frac{1}{2\sigma^2}\|\mathbf{Y} - \boldsymbol{u}\|_2^2 + \frac{1}{2\sigma^2}\|\mathbf{Y}^{\#} - \boldsymbol{u}^{\#}\|_2^2. \qquad (1)$$

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✓ The penalized likelihood ratio test statistic :

$$\Delta(\mathbf{Y}^{\bullet,\#}) = \min_{\boldsymbol{u}^{\bullet,\#}: H_0 \text{ is true}} p\ell(\mathbf{Y}^{\bullet,\#}, \boldsymbol{u}^{\bullet,\#}) - \min_{\boldsymbol{u}^{\bullet,\#}} p\ell(\mathbf{Y}^{\bullet,\#}, \boldsymbol{u}^{\bullet,\#}).$$

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- ✓ It is clear that $\Delta(\mathbf{Y}^{\bullet,\#})$ is always non-negative. Furthermore, it is small when H_0 is satisfied and is large if H_0 is violated.
- ✓ The minimization of the quadratic functional $p\ell$ leads to :

$$\min_{\boldsymbol{u}^{\bullet,\#}} p\ell(\mathbf{Y}^{\bullet,\#}, \boldsymbol{u}^{\bullet,\#}) = \frac{1}{2\sigma^2}\sum_{j\geq 1}\frac{\omega_j}{1+\omega_j}\big(|Y_j|^2 + |Y_j^{\#}|^2\big).$$

Shrinkage weights and test definition

- ✓ We replace $\omega_j/(1+\omega_j)$ by ν_j and call the sequence $\boldsymbol{\nu} = (\nu_1, \nu_2, \dots)$ shrinkage weights.
- ✓ We assume $\nu_j \in [0,1]$, for instance,

$$\nu_j = \begin{cases} \mathbb{1}_{\{j \le N_\sigma\}}, & & \text{(projection filter)} \\ \left\{1 + \left(\frac{j}{\kappa N_\sigma}\right)^{\mu}\right\}^{-1} \mathbb{1}_{\{j \le N_\sigma\}}, & \kappa > 0,\ \mu > 1, & \text{(Tikhonov filter)} \\ \left\{1 - \left(\frac{j}{N_\sigma}\right)^{\mu}\right\}_+, & \mu > 0. & \text{(Pinsker filter)} \end{cases}$$

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- ✓ The (penalized) log-likelihood ratio test is then defined by the critical region

$$\Delta_\sigma(\mathbf{Y}^{\bullet,\#}) \ge t_{\boldsymbol{\nu},\alpha},$$

  which is equivalent to

$$\boxed{\min_{\tau \in [0,2\pi]} \sum_{j \ge 1} \nu_j |Y_j - e^{\mathbf{i} j\tau} Y_j^{\#}|^2 \ge \sigma^2 t_{\boldsymbol{\nu},\alpha}.}$$

Assumptions and main results

Convergence under H_0

Main conditions on the shrinkage weights :

(**A**) $\nu_1 = 1$, and $\nu_j = 0, \ \forall j > N_\sigma$,

(B) for some positive constant $\underline{c}$, it holds that $\sum_{j \geq 1} \nu_j^2 \geq \underline{c} N_\sigma$.

Theorem

Let $\boldsymbol{c} \in \mathcal{F}_{1,L}$ and $|c_1| > 0$. Assume that the weights ν_j are chosen to satisfy conditions **(A)**, **(B)**, *$N_\sigma \to +\infty$ and $\sigma^2 N_\sigma^{5/2} \log(N_\sigma) = o(1)$. Then, under the null hypothesis, the test statistic $\Delta_\sigma(\mathbf{Y}^{\bullet,\#})$ is asymptotically distributed as a Gaussian random variable :*

$$\frac{\Delta_\sigma(\mathbf{Y}^{\bullet,\#}) - 4\|\boldsymbol{\nu}\|_1}{4\|\boldsymbol{\nu}\|_2} \xrightarrow[\sigma \to 0]{\mathscr{D}} \mathcal{N}(0,1).$$

Some consequences and remarks

- ✓ In the case of projection weights, it holds that

$$\Delta_\sigma(\mathbf{Y}^{\bullet,\#}) \overset{\mathcal{D}}{\approx} 2\chi^2_{2N_\sigma}, \qquad \sigma \to 0,$$

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- ✓ We will reject H_0 if and only if

$$\boxed{\Delta_\sigma(\mathbf{Y}^{\bullet,\#}) \geq 4\|\boldsymbol{\nu}\|_1 + 4z_{1-\alpha}\|\boldsymbol{\nu}\|_2,}$$

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- ✓ Since $c \in \mathcal{F}_{1,L}$, the "optimal" choice of weights $\boldsymbol{\nu}$ is such that $N_\sigma \sim \sigma^{-2/3}$. For this choice, the assumptions of Theorem are clearly satisfied ($\sigma^2 N_\sigma^{5/2} \log(N_\sigma) = o(1)$).

- ✓ The p-value of the aforementioned test is a measure of alignment for the pair of curves :

$$\alpha^* = \Phi\Big(\frac{\Delta_\sigma(\boldsymbol{y}^{\bullet,\#}) - 4\|\boldsymbol{\nu}\|_1}{4\|\boldsymbol{\nu}\|_2}\Big),$$

where Φ is the c.d.f. of $\mathcal{N}(0,1)$.

Behavior under the alternative

Consistency of the PLR test

We consider the following alternative

$$H_1 : \qquad \inf_{\tau} \sum_{j \geq 1} |c_j - e^{\mathbf{i}\, j\tau} c_j^{\#}|^2 \geq \rho$$

with some fixed $\rho > 0$.
We will need the following condition :

$$\textbf{(C)} \quad \exists\, \overline{c} > 0, \text{ such that } \min\{j \geq 0, \nu_j < \overline{c}\} \to +\infty, \text{ as } \sigma \to 0.$$

Theorem

Let the assumptions of the previous Theorem, as well as condition **(C)***, be satisfied. Then the test statistic* $T_\sigma = \frac{\Delta_\sigma(\mathbf{Y}^{\bullet,\#}) - 4\|\boldsymbol{\nu}\|_1}{4\|\boldsymbol{\nu}\|_2}$ *diverges under* H_1*, i.e.,*

$$T_\sigma \xrightarrow{P} +\infty, \qquad \text{as} \qquad \sigma \to 0.$$

In other words, the result above claims that the power of the PLR-test is asymptotically equal to one as the noise level σ decreases to 0.

Numerical example

In order to illustrate the convergence of the PLR-test when $\sigma \to 0$, we chose the function HeaviSine and computed its complex Fourier coefficients $\{c_j; j = 0, \dots, 10^6\}$. For each $\sigma \in \{2^{-k/2}, k = 1, \dots, 15\}$, we repeated 1000 times :

- ✓ set $N_\sigma = 50\sigma^{-1/2}$,
- ✓ generate $\{Y_j; j = 0, \dots, N_\sigma\}$ by adding to $\{c_j\}$ an i.i.d. (complex valued) sequence $\xi_j \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2)$,
- ✓ randomly choose a parameter $\tau^* \sim \mathcal{U}([0, 2\pi])$, indep. of $\{\xi_j\}$,
- ✓ generate the shifted noisy sequence $\{Y_j^{\#}; j = 0, \dots, N_\sigma\}$ by adding to $\{e^{\mathbf{i} j\tau^*} c_j\}$ an i.i.d. sequence $\xi_j^{\#} \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2)$, independent of $\{\xi_j\}$ and of τ^*,
- ✓ we compute the three values of the test statistic Δ_σ corresponding to the classical shrinkage weights and compare these values with the threshold for $\alpha = 5\%$.

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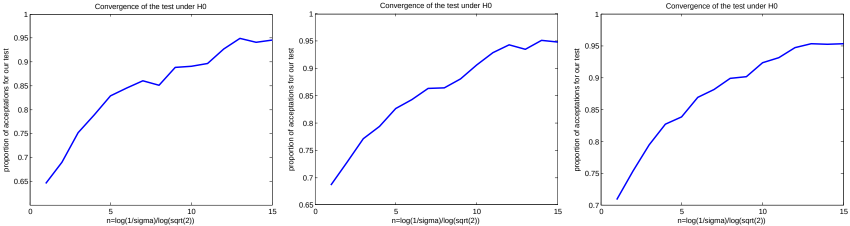

FIGURE: The proportion of acceptances as a function of $\log_2 \sigma^{-2}$ for three different shrinkage weights : projection (Left), Tikhonov-Phillips (Middle) and Pinsker (Right).

- One can observe that for $\sigma = 2^{-15/2} \approx 5 \times 10^{-3}$, the proportion of true negatives is almost equal to the nominal level 0.95.
- Another observation is that the three curves are quite comparable, with a slight advantage for the Pinsker's weights.

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- We also want to develop a direct inference for the Poisson process model presented in the beginning.
- The ultimate goal is to apply this methodology to the problem of key-point matching in computer vision.

THANK YOU !