

Poisson intensity registration by goodness-of-fit testing

with Olivier Collier

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- ◀ **Particularity** : the null hypothesis is composite and nonparametric.

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Key-point matching is a central problem in computer vision, used for object detection, tracking, stereo-vision, etc.



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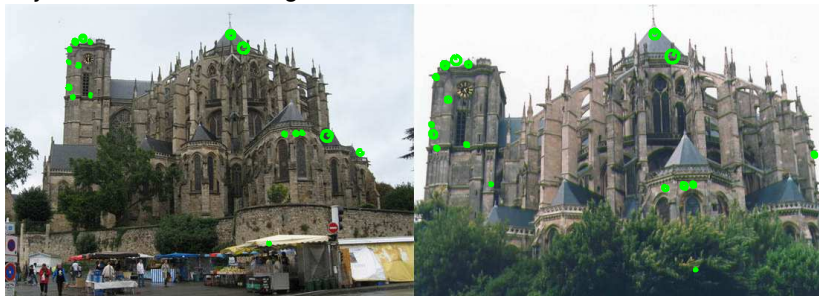
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- ◀ An image is a realization of a (2D) Poisson process.
- ◀ If in a ring, two images coincide up to a rotation, then the corresponding Poisson processes have intensities that are equal up to a shift.
- ◀ Applying rigorous statistical approach may considerably reduce the number of mismatches.

Reduction to the regression model

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- It is clear that $nX_n(A) \sim \mathcal{P}(n \int_A S(x) dx)$ and therefore

$$X_n(A) \stackrel{\mathcal{D}}{\approx} \int_A S(x) dx + \frac{1}{\sqrt{n}} \mathcal{N}\left(0, \int_A S(x) dx\right).$$

or, equivalently,

$$X_n(dx) \stackrel{\mathcal{D}}{\approx} S(x) dx + \frac{\sqrt{S(x)}}{\sqrt{n}} dB(x).$$

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From now on, we deal with the Gaussian white noise model in which :

- ◀ we observe $\mathbf{Y}^{\bullet, \#} = \{Y^{\bullet, \#}(x) = (Y(x), Y^{\#}(x)) : x \in [0, 1]\}$ s. t.

$$dY^{\bullet, \#}(x) = \begin{bmatrix} f(x) \\ f^{\#}(x) \end{bmatrix} + \sigma d\mathbf{W}(x), \quad \forall x \in [0, 1]$$

where \mathbf{W} is a 2D Brownian motion, f and $f^{\#}$ are two unknown 1-periodic signals. (We can abandon the assumption of positiveness.)

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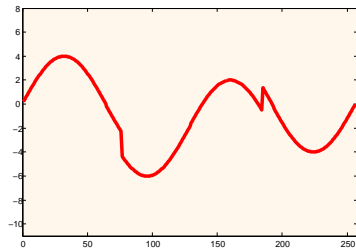
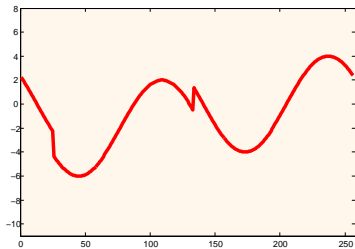
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- ◀ We wish to test

$$H_0 : \exists (b^*, \tau^*) \text{ s.t. } \boxed{f(x) = f^{\#}(x + \tau^*) + b^*}, \quad \forall x \in [0, 1].$$

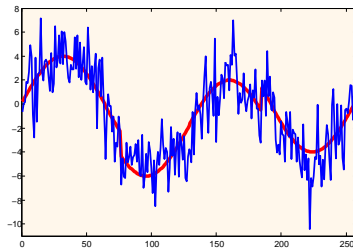
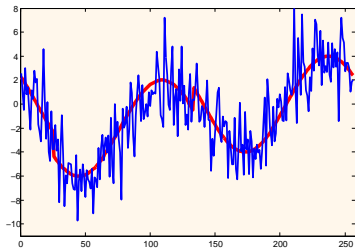
An illustration

Two periodic curves coinciding up to a time-shift



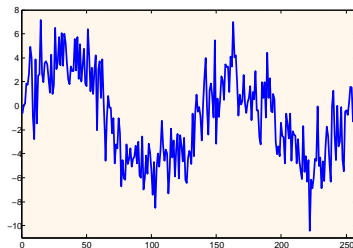
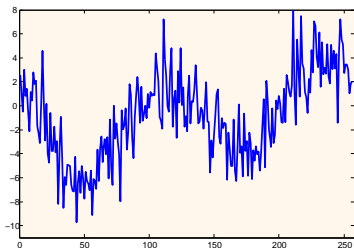
An illustration

Two periodic curves coinciding up to a time-shift and corrupted by noise



An illustration

Having observed the noisy curves



is it possible to detect that the original noiseless curves were equal up to a time shift ?

Projection onto the Fourier basis

Gaussian sequence model

- ◀ We transform the data into :

$$Y_j = c_j + \sigma \epsilon_j, \quad Y_j^\# = c_j^\# + \sigma \epsilon_j^\#, \quad j = 0, 1, 2, \dots,$$

where $c_j = \int_0^1 f(x) e^{2ij\pi x} dx$ and $c_j^\# = \int_0^1 f^\#(x) e^{2ij\pi x} dx$ are the complex Fourier coefficients.

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- ◀ We are interested in testing the hypothesis H_0 , which translates in the Fourier domain to

$$H_0 : \exists \bar{\tau}^* \in [0, 2\pi[\quad \text{s.t.} \quad c_j = e^{-ij\bar{\tau}^*} c_j^\# \quad \forall j = 1, 2, \dots$$

- ◀ The unknown parameters are assumed to belong to the functional class :

$$\mathcal{F}_{s,L} = \left\{ \mathbf{u} = (u_1, u_2, \dots) : \sum_{j=1}^{+\infty} j^{2s} |u_j|^2 \leq L^2 \right\},$$

where the positive real numbers s and L stand for the smoothness and the radius of the class $\mathcal{F}_{s,L}$.

Penalized likelihood ratio (PLR) test

✓ log-likelihood of $\mathbf{u}^{\bullet, \#} = (\mathbf{u}, \mathbf{u}^{\#})$ given $\mathbf{Y}^{\bullet, \#} = (\mathbf{Y}, \mathbf{Y}^{\#})$ is

$$\ell(\mathbf{Y}^{\bullet, \#}, \mathbf{u}^{\bullet, \#}) = \frac{1}{2\sigma^2} \|\mathbf{Y} - \mathbf{u}\|_2^2 + \frac{1}{2\sigma^2} \|\mathbf{Y}^{\#} - \mathbf{u}^{\#}\|_2^2. \quad (1)$$

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- ✓ The penalized likelihood ratio test statistic :

$$\Delta(\mathbf{Y}^{\bullet, \#}) = \min_{\mathbf{u}^{\bullet, \#}: H_0 \text{ is true}} p\ell(\mathbf{Y}^{\bullet, \#}, \mathbf{u}^{\bullet, \#}) - \min_{\mathbf{u}^{\bullet, \#}} p\ell(\mathbf{Y}^{\bullet, \#}, \mathbf{u}^{\bullet, \#}).$$

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- ✓ It is clear that $\Delta(\mathbf{Y}^{\bullet, \#})$ is always non-negative. Furthermore, it is small when H_0 is satisfied and is large if H_0 is violated.
- ✓ The minimization of the quadratic functional $p\ell$ leads to :

$$\min_{\mathbf{u}^{\bullet, \#}} p\ell(\mathbf{Y}^{\bullet, \#}, \mathbf{u}^{\bullet, \#}) = \frac{1}{2\sigma^2} \sum_{j \geq 1} \frac{\omega_j}{1 + \omega_j} (|Y_j|^2 + |Y_j^{\#}|^2).$$

Shrinkage weights and test definition

- ✓ We replace $\omega_j/(1 + \omega_j)$ by ν_j and call the sequence $\nu = (\nu_1, \nu_2, \dots)$ shrinkage weights.
- ✓ We assume $\nu_j \in [0, 1]$, for instance,

$$\nu_j = \begin{cases} \mathbb{1}_{\{j \leq N_\sigma\}}, & \text{(projection filter)} \\ \{1 + (\frac{j}{\kappa N_\sigma})^\mu\}^{-1} \mathbb{1}_{\{j \leq N_\sigma\}}, & \kappa > 0, \mu > 1, \quad \text{(Tikhonov filter)} \\ \{1 - (\frac{j}{N_\sigma})^\mu\}_+, & \mu > 0. \quad \text{(Pinsker filter)} \end{cases}$$

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- ✓ The (penalized) log-likelihood ratio test is then defined by the critical region

$$\Delta_\sigma(\mathbf{Y}^\bullet, \#) \geq t_{\boldsymbol{\nu}, \alpha},$$

which is equivalent to

$$\min_{\tau \in [0, 2\pi]} \sum_{j \geq 1} \nu_j |Y_j - e^{j\tau} Y_j^\#|^2 \geq \sigma^2 t_{\boldsymbol{\nu}, \alpha}.$$

Assumptions and main results

Convergence under H_0

Main conditions on the shrinkage weights :

(A) $\nu_1 = 1$, and $\nu_j = 0, \forall j > N_\sigma$,

(B) for some positive constant \underline{c} , it holds that $\sum_{j \geq 1} \nu_j^2 \geq \underline{c} N_\sigma$.

Theorem

Let $\mathbf{c} \in \mathcal{F}_{1,L}$ and $|c_1| > 0$. Assume that the weights ν_j are chosen to satisfy conditions (A), (B), $N_\sigma \rightarrow +\infty$ and $\sigma^2 N_\sigma^{5/2} \log(N_\sigma) = o(1)$. Then, under the null hypothesis, the test statistic $\Delta_\sigma(\mathbf{Y}^\bullet, \#)$ is asymptotically distributed as a Gaussian random variable :

$$\frac{\Delta_\sigma(\mathbf{Y}^\bullet, \#) - 4\|\boldsymbol{\nu}\|_1}{4\|\boldsymbol{\nu}\|_2} \xrightarrow[\sigma \rightarrow 0]{\mathcal{D}} \mathcal{N}(0, 1).$$

Some consequences and remarks

- ✓ In the case of projection weights, it holds that

$$\Delta_{\sigma}(\mathbf{Y}^{\bullet, \#}) \stackrel{\mathcal{D}}{\approx} 2\chi_{2N_{\sigma}}^2, \quad \sigma \rightarrow 0,$$

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- ✓ We will reject H_0 if and only if

$$\Delta_{\sigma}(\mathbf{Y}^{\bullet, \#}) \geq 4\|\boldsymbol{\nu}\|_1 + 4z_{1-\alpha}\|\boldsymbol{\nu}\|_2,$$

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- ✓ Since $\mathbf{c} \in \mathcal{F}_{1,L}$, the “optimal” choice of weights $\boldsymbol{\nu}$ is such that $N_{\sigma} \sim \sigma^{-2/3}$. For this choice, the assumptions of Theorem are clearly satisfied ($\sigma^2 N_{\sigma}^{5/2} \log(N_{\sigma}) = o(1)$).
- ✓ The p -value of the aforementioned test is a measure of alignment for the pair of curves :

$$\alpha^* = \Phi\left(\frac{\Delta_{\sigma}(\mathbf{y}^{\bullet, \#}) - 4\|\boldsymbol{\nu}\|_1}{4\|\boldsymbol{\nu}\|_2}\right),$$

where Φ is the c.d.f. of $\mathcal{N}(0, 1)$.

Behavior under the alternative

Consistency of the PLR test

We consider the following alternative

$$H_1 : \inf_{\tau} \sum_{j \geq 1} |c_j - e^{j\tau} c_j^{\#}|^2 \geq \rho$$

with some fixed $\rho > 0$.

We will need the following condition :

(C) $\exists \bar{c} > 0$, such that $\min\{j \geq 0, \nu_j < \bar{c}\} \rightarrow +\infty$, as $\sigma \rightarrow 0$.

Theorem

Let the assumptions of the previous Theorem, as well as condition **(C)**, be satisfied. Then the test statistic $T_{\sigma} = \frac{\Delta_{\sigma}(\mathbf{Y}^{\bullet, \#}) - 4\|\nu\|_1}{4\|\nu\|_2}$ diverges under H_1 , i.e.,

$$T_{\sigma} \xrightarrow{P} +\infty, \quad \text{as } \sigma \rightarrow 0.$$

In other words, the result above claims that the power of the PLR-test is asymptotically equal to one as the noise level σ decreases to 0.

Numerical example

In order to illustrate the convergence of the PLR-test when $\sigma \rightarrow 0$, we chose the function HeaviSine and computed its complex Fourier coefficients $\{c_j; j = 0, \dots, 10^6\}$. For each $\sigma \in \{2^{-k/2}, k = 1, \dots, 15\}$, we repeated 1000 times :

- ✓ set $N_\sigma = 50\sigma^{-1/2}$,
- ✓ generate $\{Y_j; j = 0, \dots, N_\sigma\}$ by adding to $\{c_j\}$ an i.i.d. (complex valued) sequence $\xi_j \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \sigma^2)$,
- ✓ randomly choose a parameter $\tau^* \sim \mathcal{U}([0, 2\pi])$, indep. of $\{\xi_j\}$,
- ✓ generate the shifted noisy sequence $\{Y_j^\#; j = 0, \dots, N_\sigma\}$ by adding to $\{e^{ij\tau^*} c_j\}$ an i.i.d. sequence $\xi_j^\# \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \sigma^2)$, independent of $\{\xi_j\}$ and of τ^* ,
- ✓ we compute the three values of the test statistic Δ_σ corresponding to the classical shrinkage weights and compare these values with the threshold for $\alpha = 5\%$.

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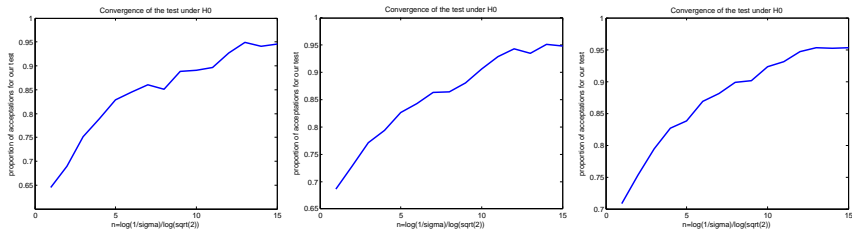


FIGURE: The proportion of acceptances as a function of $\log_2 \sigma^{-2}$ for three different shrinkage weights : projection (Left), Tikhonov-Phillips (Middle) and Pinsker (Right).

- ▶ One can observe that for $\sigma = 2^{-15/2} \approx 5 \times 10^{-3}$, the proportion of true negatives is almost equal to the nominal level 0.95.
- ▶ Another observation is that the three curves are quite comparable, with a slight advantage for the Pinsker's weights.

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- ▶ We are now close to completing the proof of the minimax rates of separation, in the spirit of Ingster and Kutoyants [MMS, 2007].
- ▶ We also want to develop a direct inference for the Poisson process model presented in the beginning.
- ▶ The ultimate goal is to apply this methodology to the problem of key-point matching in computer vision.

THANK YOU !