The law of the iterated logarithm and data-driven regularizations of ill-posed inverse problems

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Abstract

The talk deals with recovering an unknown function $\theta(t)$, $t \in [0,1]$ from the noisy data $dY(t) = \int_0^1 A(t,u)\theta(u)\,du\,dt + \sigma\,dW(t)$, $t \in [0,1]$, where $A(t,u)$ is a known function with the bounded L_2 - norm and $W(\cdot)$ is a standard Wiener process. In order to estimate θ, a spectral regularization method is used, and our goal is to choose its regularization parameter with the help of the data at hand. For a very wide class spectral regularization methods, we propose a new method for computing penalties in the principle of empirical risk minimization and show that the third order term in nearly optimal penalties is related to a non-asymptotic version of the law of the iterated logarithm. It is shown that this approach results in sharp oracle inequalities controlling the mean square risks of data-driven spectral regularization methods.