

## A limit theorem for likelihoods in the LAQ case

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(joint work with Esko Valkeila)

### Abstract

We consider a general sequence  $(\Omega^n, \mathcal{F}^n, (\mathcal{F}_t^n)_{t \in \mathbb{R}_+}, (\mathbf{P}^{n, \vartheta})_{\vartheta \in \Theta})$  of filtered statistical experiments, where  $\Theta$  is an open subset of  $\mathbb{R}^k$ . Given a normalizing sequence  $\varphi_n \rightarrow 0$  of positive definite  $k \times k$  matrices, we are interested in limit theorems for the likelihood processes of  $\mathbf{P}^{n, \vartheta_0 + \varphi_n \vartheta}$  with respect to  $\mathbf{P}^{n, \vartheta_0}$  in the case where the limiting likelihood process is of the form  $\exp(\vartheta^\top M - \frac{1}{2} \vartheta^\top \langle M \rangle \vartheta)$ ,  $M$  being a continuous local martingale (not necessarily Gaussian or conditionally Gaussian). General limit theorems for likelihood processes, corresponding to such a limit, e.g. Theorems X.1.59 and X.1.65 in Jacod and Shiryaev (1987) and Theorem 3.9 in Gushchin and Valkeila (2003), have certain drawbacks. First, the majoration conditions X.1.57 c) in Jacod and Shiryaev (1987) and M c) in Gushchin and Valkeila (2003) are quite restrictive. Second, in many models the process  $M$  happens to be of the form  $\int K dB$ , where  $B$  is a Brownian motion and  $K$  is adapted with respect to the filtration generated by  $B$ . Thus, it is inconvenient to represent the quadratic characteristic of  $M$  as a functional of  $M$  or of the likelihood process as is needed in the above-mentioned theorems.

The aim of the talk is to present a theorem avoiding these drawbacks. Though a majoration condition is still imposed, it refers to another process which is a Gaussian martingale in many applications, and then the condition is trivially satisfied. Some examples are considered.