On exceptional properties of f-divergence minimal martingale measures for exponential Levy models

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Plan

Exponential Levy models

Exponential Levy models have been widely used since the 1990's to represent asset prices.

- classical Black-Scholes model
- Generalized Hyperbolic models
- Variance-Gamma models

This class is flexible enough to model well one-dimensional distributions of stock prices.

Exponential Levy models

- $X = (X_t)_{0 \le t \le T}$ is d-dimensional Levy process with parameters (b, c, ν)
- $S_t = \exp(X_t)$
- $Q \overset{loc}{\sim} P$
- S is a martingale under Q iff the drift of S is equal to zero.

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Minimisations

- risk-minimisation in an L^2-sense (Follmer, Schweizer; Schweizer)
- Hellinger integrals minimisation (Choulli, Stricker; Choulli, Stricker, Li)
- entropy minimisation (Miyahara; Fujiara, Miyahara; Grandits; Rheinländer)
- f^q- minimisation(Jeanblanc, Klöppel, Myahara)
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f-divergence

- P and Q two probability measures, $Q << P$
- f is a convex function on $\mathbb{R}^{+,*}$
- f-divergence is
$$f(Q|P) = \mathbb{E}_P[f(\frac{dQ}{dP})]$$
- In particular cases, when $f(x) = -x^\alpha, 0 < \alpha < 1$ we obtain Hellinger integral, when $f(x) = x\ln(x)$ we obtain entropy, with $f(x) = (1-x)^2$ we have squared variance distance, with $f(x) = |1-x|$ we have variance distance.

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Some important properties

- We say that Q^* is an f-divergence minimal martingale measure if $f(Q^*|P) < \infty$ and

$$f(Q^*|P) = \inf_{Q \in \mathcal{M}(P)} f(Q|P)$$

where $\mathcal{M}(P)$ is the set of equivalent martingale measures.

- For a given exponential Levy model $S = S_0\, e^X$, we say that an f-divergence minimal martingale measure Q^* preserves the Levy property if X remains a Levy process under Q^*.
- The preservation of Levy property under Q^* is equivalent to the fact that the Girsanov parameters (β, Y) for the change of the measure P into Q^* are independent on (t, ω).

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- The measure Q^* is said to be scale invariant if for all $x \in \mathbb{R}^+$, $f(xQ^* \| P) < \infty$ and

$$f(xQ^* \| P) = \min_{Q \in \mathcal{M}} f(xQ \| P).$$

- We also recall that an equivalent martingale measure Q^* is said to be time invariant if for all $T > 0$, and the restrictions Q_T,P_T of the measures P, Q on time interval $[0, T]$, $f(Q^*_T \| P_T) < \infty$ and

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Preservation of Levy property

- If f is classical i.e. $f''(x) = ax^{\gamma}$, $a > 0, \gamma \neq 0$ then Q^*, when exists, preserves Levy property for all Levy processes. It is also scale and time invariant.
- If Q^* exists and preserve Levy property, then, in general, f is not arbitrary. What can we say about f? Is it only classical: $f''(x) = ax^{\gamma}$?

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Notations

- Let f be strictly convex $C^3(\mathbb{R}^{+,*})$ function.
- Let Z^* be the density of an f-divergence minimal measure Q^* on $[0, T]$, which preserves the Levy property
- We denote by (β^*, Y^*) its Girsanov parameters.

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Main equality for f

Theorem

If $\overset{\circ}{supp}\,(\nu) \neq \emptyset$,
for a.e. $x \in supp(Z^*_T)$ *and a.e.* $y \in supp(\nu)$, *we have*

$$f'(xY^*(y)) - f'(x) = \Phi(x) \sum_{i=1}^{d} \alpha_i (e^{y_i} - 1)$$

where

- Φ *is a continuously differentiable function on* $\overset{\circ}{supp}\,(Z^*_T)$
- $\alpha = {}^{\top}(\alpha_1, \alpha_2, \cdots \alpha_d)$ *is a vector of* $\mathbb{R}^d$.

Furthermore, if $c \neq 0$,
for a.e. $x \in \text{supp}(Z_T^*)$ and a.e. $y \in supp(\nu)$, we have

$$f'(xY^*(y)) - f'(x) = xf''(x) \sum_{i=1}^{d} \beta_i^*(e^{y_i} - 1) - \sum_{j=1}^{d} V_j(e^{y_j} - 1)$$

where

- $\beta^* =^\top (\beta_1^*, \cdots, \beta_d^*)$
- $V =^\top (V_1, \cdots, V_d)$ belongs to the kernel of the matrix c, i.e. $cV = 0$.

Support of Z_T^*

Theorem

Let Z_T^* be the density of an f-divergence minimal equivalent martingale measure on $[0, T]$, which preserves the Levy property. Then

(i) *If* ${}^\top\beta^* c \beta^* \neq 0$, *then* $supp(Z_T^*) = \mathbb{R}^{+,*}$.

(ii) *If* ${}^\top\beta^* c \beta^* = 0$, $\overset{\circ}{supp}(\nu) \neq \emptyset$, $0 \in supp(\nu)$ *and* Y^* *is not identically 1 on* $\overset{\circ}{supp}(\nu)$, *then* $supp(Z_T^*)$ *contains an interval.*

Final result on preservation

Theorem

Let $f : \mathbb{R}^{+*} \to \mathbb{R}$ be a strictly convex function of class $\mathcal{C}^3$ and let X be a Levy process given by its characteristics (b, c, ν). Assume there exists an f-divergence minimal martingale measure Q^* on a time interval $[0, T]$, which preserves the Levy property. Let also some integrability conditions be verified.

If the set $\{\ln Y^*(y),\, y \in supp(\nu)\}$ is of the non-empty interior and it contains zero, then there exists $a > 0$ and $\gamma \in \mathbb{R}^*$ such that for all $x \in supp(Z_T^*)$,

$$f''(x) = a x^{\gamma}.$$

If ${}^{\top}\beta^* c \beta^* \neq 0$ and there exists $y \in supp(\nu)$ such that $Y^*(y) \neq 1$,

then there exist $n \in \mathbb{N}$, $\gamma \in \mathbb{R}$, $a > 0$ and the real constants $b_i, \tilde{b}_i, 1 \leq i \leq n$, such that

$$f''(x) = a x^{\gamma} + x^{\gamma} \sum_{i=1}^{n} b_i (\ln(x))^i + \frac{1}{x} \sum_{i=1}^{n} \tilde{b}_i (\ln(x))^{i-1}$$

Example

- W is a standard BM, P is standard Poisson process
- $X_t = {}^{\top}(W_t + \ln 2\, P_t, W_t + \ln 3\, P_t - t)$
- $f(x) = x^2/2 + x \ln x$
- There exists f-minimal equivalent martingale measure preserving Levy property.

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Goll, Ruschendorf result

Theorem

Let $x \in \mathbb{R}^+$ be fixed. Let Q^* be an equivalent f-divergence minimal martingale measure which satisfies

$$\mathbb{E}_P[f(\lambda \frac{dQ_T^*}{dP_T})] < \infty, \;\; \mathbb{E}_{Q^*}|f'(\lambda \frac{dQ_T^*}{dP_T})| < \infty$$

with λ such that

$$-\mathbb{E}_{Q^*} f'(\lambda \frac{dQ_T^*}{dP_T}) = x.$$

Goll, Ruschendorf result

Then

$$-f'(\lambda \frac{dQ^*_T}{dP_T}) = x + \int_0^T \phi_u dS_u$$

where ϕ is predictable function such that $(\int_0^{\cdot} \phi_u dS_u)$ is Q^*-martingale.

If the last relation holds, then $\overrightarrow{\Phi} = (\phi^0, \phi)$ with

$$\phi^0_t = x + \int_0^t \phi_u dS_u - \phi_t S_t$$

is an admissible optimal portfolio strategy.

Decomposition formula for conditional expectation

We introduce càdlàg versions of the processes $(\xi_t(x))_{t\geq 0}$ et $(H_t(x,y))_{t\geq 0}$ where for $0 \leq t \leq T$

- $\xi_t(x) = E_Q[f''(xZ_{T-t})Z_{T-t}]$
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Theorem

Let f be a strictly convex function belonging to $C^3(\mathbb{R}^{+,*})$. Let Z be the density of a Levy preserving equivalent martingale measure Q. Then, for all $\lambda > 0$ we have Q- a.s, for all $t \leq T$,

$$E_Q[f'(\lambda Z_T)|\mathcal{F}_t] =$$

$$E_Q[f'(\lambda Z_T)] + \sum_{i=1}^{d} \lambda\beta^{(i)} \int_0^t \xi_s(\lambda Z_{s-})\, Z_{s-}\, dX_s^{(c),Q,i}$$

$$+ \int_0^t \int_{\mathbb{R}^d} H_s(\lambda Z_{s-}, y)\, (\mu^X - \nu^{X,Q})(ds, dy)$$

Approximations

Lemma

Let f be a strictly convex function belonging to $C^3(\mathbb{R}^{+,*})$. There exists a sequence of bounded increasing functions $(\phi_n)_{n\geq 1}$, which are of class $\mathcal{C}^2$ on $\mathbb{R}^{+,*}$, such that for all $n \geq 1$, ϕ_n coincides with f' on the compact set $[\frac{1}{n}, n]$ and such that for n large enough and for all $x, y > 0$ the following inequalities hold :

- $|\phi_n(x)| \leq 4|f'(x)| + \alpha$ *where* α *is a real positive constant.*
- $|\phi'_n(x)| \leq 3f''(x)$
- $|\phi_n(x) - \phi_n(y)| \leq 5|f'(x) - f'(y)|$

Utility function

- two assets : a non-risky asset B, with interest rate r, and a risky asset $S = S_0 e^X$ on filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, P)$
- $\vec{S} = (B, S)$ is the price process and $\vec{\Phi} = (\phi^0, \phi)$ is strategy
- A predictable $\vec{S}$-integrable process $\vec{\Phi}$ will be said to be a self-financing admissible strategy if for every $t \in [0, T]$ and x initial capital

$$\vec{\Phi}_t \cdot \vec{S}_t = x + \int_0^t \vec{\Phi}_u \cdot d\vec{S}_u$$

where the stochastic integral in the right-hand side is bounded from below.

- if the interest rate r is 0, then terminal wealth at time T is

$$V_T(\phi) = x + \int^T \phi_s dS_s$$

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Optimal strategies

Let u denote a strictly increasing, strictly concave, continuously differentiable function on $dom(u) = \{x \in \mathbb{R} | u(x) > -\infty\}$ which satisfies

$$u'(+\infty) = \lim_{x \to +\infty} u'(x) = 0,$$

$$u'(\underline{x}) = \lim_{x \to \underline{x}} u'(x) = +\infty$$

where $\underline{x} = \inf\{u \in dom(u)\}$.

Utility maximizing strategy ϕ^*:

$$\sup_{\phi \in \mathcal{A}} \mathbb{E}_P(u(V_T(\phi))) = \mathbb{E}_P(u(V_T(\phi^*)))$$

Theorem

Let u be a $\mathcal{C}^3(]\underline{x},+\infty[)$ utility function and f its convex conjugate. Assume there exists an f-minimal martingale measure Q^* which preserves the Levy property and such that some integrability conditions are satisfied.

Then for any fixed initial capital $x > \underline{x}$, there exists an asymptotically u-optimal strategy $\hat{\phi}$. In addition, $\hat{\phi}$ defines a u-optimal strategy as soon as $\underline{x} > -\infty$.

Furthermore, if $c \neq 0$, we have

$$\hat{\phi}_s^{(i)} = -\frac{\lambda \beta^{(i)} Z_{s-}^*}{S_{s-}^{(i)}} \xi_s(\lambda Z_{s-}^*)$$

where

- $\beta =^\top (\beta^{(1)}, \cdots, \beta^{(d)})$ is the first Girsanov parameter,
- $\xi_s(\cdot) = E_{Q^*}[f''(xZ_{T-s}^*)Z_{T-s}^*]$,
- λ is a unique solution to the equation $E_{Q^*}(-f'(\lambda Z_T^*)) = x$.

If $c = 0$, $\overset{\circ}{supp}\,(\nu) \neq \emptyset$ and it contains zero and Y^* is not identically 1, then $f''(x) = ax^\gamma$ with $a > 0$ and $\gamma \in \mathbb{R}^*$, and

$$\hat{\phi}_s^{(i)} = -\frac{\lambda\gamma^{(i)} Z^*_{s-}}{S^{(i)}_{s-}}\, \xi_s(\lambda Z^*_{s-})$$

where again λ is a unique solution to the equation

$$E_{Q^*}(-f'(\lambda Z^*_T)) = x$$

and the constants $\gamma^{(i)}$ are related with the second Girsanov parameter Y by the formula:

$$\gamma^{(i)} = \exp(-y_{0,i})\, Y(y_0)^\gamma \,\frac{\partial}{\partial y_i} Y(y_0)$$

Classical utilities

Theorem

Consider a Levy process X with characteristics (b, c, ν) and let f be a function such that $f''(x) = ax^{\gamma}$, where $a > 0$ and $\gamma \in \mathbb{R}^*$. Let u_f be its concave conjugate. Assume there exist $\alpha, \beta \in \mathbb{R}^d$ and a measurable function $Y : \mathbb{R}^d \setminus \{0\} \to \mathbb{R}^+$ such that

$$Y(y) = (f')^{-1}(f'(1) + \sum_{i=1}^{d} \alpha^{(i)}(e^{y_i} - 1))$$

Suppose that the following properties hold:

$$Y(y) > 0 \;\; \nu - a.e.,$$

$$\sum_{i=1}^{d} \int_{|y|\geq 1} (e^{y_i} - 1) Y(y) \nu(dy) < +\infty.$$

$$b + \frac{1}{2} diag(c) + c\beta + \int_{\mathbb{R}^d} ((e^y - 1) Y(y) - h(y)) \nu(dy) = 0.$$

Then, if $c \neq 0$, there exists an asymptotically optimal strategy $\hat{\phi}$ whose coordinates are given by

$$\hat{\phi}_s^{(i)} = \alpha_{\gamma+1}(x) \frac{\beta^{(i)}}{E_P[Z_s^{\gamma+2}]} \frac{Z_{s-}^{\gamma+1}}{S_{s-}^{(i)}},$$

where

$$\alpha_{\gamma+1}(x) = -(\gamma+1)(x + f'(1)) + a.$$

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In addition, in both cases $\hat{\phi}$ is optimal as soon as $\gamma \neq -1$.