Continuity of stationary solutions of delay differential equations driven by Lévy processes

Alexander Gushchin (Steklov Mathematical Institute, Moscow, Russia)
(joint work with Oleg Butkovsky (Technion, Haifa, Israel))

Abstract

We consider a one-dimensional affine stochastic delay differential equation

$$dX(t) = \int_{[-r,0]} X(t+s)\,\lambda(ds)\,dt + dL(t), \quad t \geq 0,$$
$$X(t) = X_0(t), \quad -r \leq t \leq 0,$$

where $r > 0$, λ is a finite signed measure on $[-r, 0]$, L is a one-dimensional Lévy process with a Gaussian component and a finite logarithmic moment, and $X_0 = \{X_0(s),\, -r \leq s \leq 0\}$ is the initial condition independent of L. Define, as usual, $X_t := \{X(t+s),\, -r \leq s \leq 0\}$, $t \geq 0$. This equation has a unique strong solution and the process $X := (X_t)_{t \geq 0}$ is a homogeneous Markov process taking values in the Skorokhod space $\mathbb{D} = \mathbb{D}([-r, 0], \mathbb{R})$. Denote by $\mathbb{M}^-$ the subset of finite signed measures on $[-r, 0]$ such that the equation

$$z - \int_{[-r,0]} \mathrm{e}^{zs} a(ds) = 0$$

has no roots in the right half complex plane $\{z \in \mathbb{C}\colon \operatorname{Re} z \geq 0\}$. It was proved in Gushchin and Küchler (2000) that if $\lambda \in \mathbb{M}^-$. then there is a (unique in law) stationary solution of the above equation. We denote the corresponding invariant distribution by $\pi(\lambda)$.

The main purpose of the talk is to show that if λ_n, $\lambda \in \mathbb{M}^-$, and the sequence λ_n weakly converges to λ, then the sequence $\pi(\lambda_n)$ converges to $\pi(\lambda)$ in the total variation norm. This question is important for asymptotic parametric inference concerning the delay measure λ.