

**Harris recurrence for strongly degenerate stochastic systems,
with application to stochastic Hodgkin-Huxley models**

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Abstract

We consider strongly degenerate stochastic differential equations having analytic coefficients, a diffusion coefficient which does not depend on time, and a drift depending on both time and spatial position which is periodic in the time argument. Our aim is to give simple criteria for positive Harris recurrence. These are formulated in terms of control systems and the support theorem, in terms of one inner point of the state space which is of full weak Hörmander dimension, and in terms of some Lyapunov function.

As an application, we can consider a stochastic Hodgkin-Huxley model for a spiking neuron where dendritic input –carrying some deterministic periodic signal coded in its drift coefficient– is the only source of noise. This amounts to a 5d SDE driven by 1d Brownian motion for which we can prove positive Harris recurrence. This approach provides us with laws of large numbers which allow to describe the spiking activity of the neuron in the long run.