On the estimation of analytic intensity density Poisson random fields

Ibragimov Ildar (St. Petersburg Dept. Steklov Math. Institute, St. Petersburg, Russia)

Abstract

Let Π_ε be a Poisson random field with an intensity density $\frac{\lambda((x)}{\varepsilon}$ with respect to the Lebesgue measure μ in R^d. The statistician observes a Poisson random set Π_ε with un unknown function $\lambda(x)$, $\lambda \in \Lambda$ is a known set of functions, ε is a small known parameter. The problem is to estimate λ and it looks as follows. The random field Π_ε is observed in a bounded region $G \subset R^d$. It is supposed that the unknown function λ belongs to a known class of entire analytic functions. The problem is to estimate the value $\lambda(x)$ at the points $x \notin G$. We consider an asymptotic set up of the problem when $\varepsilon \to 0$.

1. We study how far from the region G the consistent estimation of λ is yet possible. The distance behave itself as $(\ln 1/\varepsilon)^a$, the power a depends on the class Λ.

2. We study how the restrictions on the region of observation influence on the rate of estimates. Here is an example of such influence. Denote $\Lambda(K)$ the class of densities λ under the following conditions:

$$\int_{R^d} \lambda(x)dx \leq 1;$$

$$\lambda(x) = \int_K e^{i(t,x)}\psi(t)dt, \ \psi \in L_2(R^d).$$

K is a bounded symmetric convex body in R^d. Then the first result says:

I. The following relation is true when $\varepsilon \to 0$:

$$\inf_{\lambda_\varepsilon} \sup_{\lambda \in \Lambda(K)} \mathbf{E}_\lambda\{||\lambda - \lambda_\varepsilon||_2^2\} \sim \varepsilon \frac{\mu(K)}{(2\pi)^d}.$$

The second result says:

Let $\lambda \in \mathbf{\Lambda}(K)$. Let G be a bounded region in R^d. The following inequalities have place

$$c_1\varepsilon\Big|\frac{\ln\varepsilon}{\ln|\ln\varepsilon|}\Big|^d \leq \inf_{\hat\lambda} \sup_{\lambda \in \mathbf{\Lambda}(K)} \int_G |\lambda(x) - \hat\lambda(x)|^2 dx \leq c_2\varepsilon\Big|\frac{\ln\varepsilon}{\ln|\ln\varepsilon|}\Big|^d$$

The low bound is taken over all estimates $\hat\lambda$ constructed on the base of the observations $\Pi_\varepsilon G$, the constants $c_1 \leq c_2$ are positive and depend on d, K, G.