Sequential parameter estimators with guaranteed accuracy for delay differential equations

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Abstract

Assume the process $(X(t), t \geq 0)$, defined by the delay differential equation

$$dX(t) = (aX(t) + bX(t-1))dt + \sigma dW(t)$$

and the initial condition

$$X(s) = x_0(s), s \in [-1, 0],$$

can be observed, where the parameters $\vartheta^* = (a, b)$ are unknown. Let ϵ be an arbitrary positive number and $q \geq 2$ fixed. A sequential estimation plan $(T_\epsilon, \hat{\vartheta}_\epsilon)$ will be presented, such that $||\hat{\vartheta}_\epsilon - \vartheta||_q \leq \epsilon$,
where $||.||_q = (E_\vartheta ||.||^q)^{\frac{1}{q}}$ and $||a||^2 = a_1^2 + a_2^2$ for $a = (a_1, a_2)^*$.
Asymptotic properties of $(T_\epsilon, \vartheta_\epsilon)$ for $\epsilon \to 0$ are studied.

The construction can be generalized to linear regression processes described by

$$dX(t) = \vartheta^* a(t)dt + \sigma dW(t)$$

based on observation of $(X(t), a(t), t \geq 0)$.