Cramér-von Mises test for Gaussian distribution in Hilbert Space

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Abstract

In statistical applications it is often assumed that an observed random process is Gaussian. Often, this assumption is accepted in practice without an adequate analysis.We propose here a generalization of the Cramér-von Mises test for testing the null hypothesis that an observed random process on the interval $[0, 1]$ is a mean zero Gaussian process with specified covariance function. We assume the alternative random processes are all the other Gaussian processes, together with all non-Gaussian processes. The test statistic is based on the finite number of the process observations. In fact, we consider the more general problem of testing the hypothesis that the distribution in a separable Hilbert space is Gaussian. To test this simple hypothesis we propose a Cramér-von Mises test based on an infinite-dimensional analogue of the empirical process. The proposed test is asymptotically consistent against all alternatives. We also provide a method for computing the critical values of our test statistic. It was calculated the exact critical values of the test. The same theory also applies to the problem of testing multivariate uniformity over a high-dimensional hypercube. This investigation is based upon previous joint work by Paul Deheuvels and the author (2003). The first idea of this method have been proposed by the author (1979, 1992). The main results are presented in the paper (2015).

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