Hybrid multi-step estimation of the volatility for stochastic regression models

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Abstract

We treat parametric estimation of the volatility for a stochastic regression model specified by the stochastic integral equation

$$Y_t = Y_0 + \int_0^t b_s ds + \int_0^t \sigma(X_s, \theta) dw_s, \quad t \in [0, T], \tag{1}$$

where w is an r-dimensional standard Wiener process on a stochastic basis $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, P)$, b and X are progressively measurable processes with values in $\mathbb{R}^m$ and $\mathbb{R}^d$, respectively, Y_0 is an $\mathbb{R}^m$-valued initial condition, σ is an $\mathbb{R}^m \otimes \mathbb{R}^r$-valued function defined on $\mathbb{R}^d \times \Theta$, and Θ is a bounded domain in $\mathbb{R}^p$. The data are discrete observations $\mathbf{Z}_n = (X_{t_k}, Y_{t_k})_{0\le k\le n}$ with $t_k = kh$ for $h = h_n = T/n$. Note that the process b is completely unobservable and unknown. The asymptotics will be considered in the situation where $n \to \infty$, which means that $\mathbf{Z}_n$ are high frequency data. Uchida and Yoshida (2013, SPA) showed that both the ML and Bayes type estimators have asymptotic normality with convergence of moments for the stochastic regression models. However, from a computational point of view, numerical optimization is necessary to get the ML-type estimators and it takes much time to compute the Bayes type estimators. Although the one-step estimator is very efficient, it is difficult to implement the one-step estimation for diffusion type processes since it is not easy to find the initial estimator with $\sqrt{n}$-consistency.

Recently, Kamatani and Uchida (2014, to appear in SISP) considered the multi-step estimation of both drift and volatility parameters for ergodic diffusion processes based on sampled data. The method can be applied to parametric inference of non-ergodic diffusion type processes from the high frequency data observed on the fixed interval. For that reason, in this talk, we propose hybrid multi-step estimators with the initial Bayes type estimator and show that the multi-step estimators have asymptotic mixed normality and convergence of moments.

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