On asymptotically efficient estimation in partially observed systems

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Abstract

We study a model related to partially observed linear systems, where the function we would like to control is not observed directly. The objective is to perform estimation of different functional characteristics of the underlying model.

Assume that we observe a process $X = (X_t,\ 0 \le t \le T)$ satisfying the following system of stochastic differential equations:

$$\begin{aligned} \mathrm{d}X_t &= h_t Y_t\, \mathrm{d}t + \varepsilon\, \mathrm{d}W_t, \quad X_0 = 0, \\ \mathrm{d}Y_t &= g_t Y_t\, \mathrm{d}t + \varepsilon\, \mathrm{d}V_t, \quad Y_0 = y_0 \neq 0, \quad 0 \le t \le T, \end{aligned}$$

where W_t and V_t, $0 \le t \le T$, are two independent Wiener processes. The process $Y = (Y_t,\ 0 \le t \le T)$ cannot be observed directly, but it is the one *we would like to control*.

In this model, we consider the problem of asymptotically efficient estimation of different functions on $0 \le t \le T$ under a *small noise*, i.e., as $\varepsilon \to 0$. We propose kernel-type estimators for the functions $f_t := h_t y_t$, h_t, y_t, g_t, $0 \le t \le T$, and study their properties. Here y_t, $0 \le t \le T$, stands for the solution of the above model given that both noise terms are dropped.

Lower bounds on the rate of convergence of asymptotically efficient estimators are obtained. We show explicitly at what estimators these lower bounds are attained.

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