Effective Discretization of Stochastic Differential Equations

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Abstract

The aim of this study is to find a generic method for generating a path of the solution of a given stochastic differential equation which is more effective than the Euler-Maruyama scheme with equidistant time partition. First we characterize the asymptotic distribution of pathwise error in the Euler-Maruyama scheme with a general partition by stopping times and then, show that the use of the hitting time of a fixed sphere for the driving Brownian motion results in a smaller error than the usual time-equidistant one with reduction ratio $d/(d+2)$, where d is the dimension of the Brownian motion. This is optimal in a stationary and symmetric class of stopping times. Next we show that the use of a specific hitting time of a moving sphere which is easy to sample results in a smaller error than the usual one as well and the reduction is close to the best possible.