The Euler-Maruyama approximations for the CEV model

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Abstract

The CEV model is given by the stochastic differential equation $X_t = X_0 + \int_0^t \mu X_s ds + \int_0^t \sigma (X_s^+)^p dW_s$, $\frac{1}{2} \le p < 1$. It features a non-Lipschitz diffusion coefficient and gets absorbed at zero with a positive probability. We show the weak convergence of Euler-Maruyama approximations X_t^n to the process X_t, $0 \le t \le T$, in the Skorokhod metric, by giving a new approximation by continuous processes. We calculate ruin probabilities as an example of such approximation. The ruin probability evaluated by simulations is not guaranteed to converge to the theoretical one, because the limiting distribution is discontinuous at zero. To approximate the size of the jump at zero we use the Levy metric, and also confirm the convergence numerically.