Local asymptotic mixed normality property for nonsynchronously observed diffusion processes

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Abstract

We study parametric estimation for diffusion processes, where observations of processes are nonsynchronous. Let $\{Y_t\} = \{(Y_t^1, Y_t^2)\}$ be a two-dimensional diffusion process which satisfy the following sthochastic differential equation :

$$dY_t = \mu(t, Y_t, \sigma)dt + b(t, Y_t, \sigma)dW_t, \quad t \in [0, T], \tag{1}$$

where $\sigma \in \Lambda$ is a parameter, $\Lambda \subset \mathbb{R}^d$ is a open set, $\{W_t\}_{0 \leq t \leq T}$ is a two-dimensional standard Wiener process and time end $T > 0$ is fixed. Let observation times $\{S^{n,i}\}_{n \in \mathbb{N}, 0 \leq i \leq l_n}$, $\{T^{n,j}\}_{n \in \mathbb{N}, 0 \leq j \leq m_n}$ of Y^1,Y^2, respectively, be stochastic, independent of $(Y_t, W_t)_{t \geq 0}$ and satisfy $\max_{i,j}(|S^i - S^{i-1}| \vee |T^j - T^{j-1}|) \to^p 0$ as $n \to \infty$. Let $P_{\sigma,n}$ be the distribution of $\{S^{n,i}\}$, $\{T^{n,j}\}$, $\{Y^1_{S^{n,i}}\}$, $\{Y^2_{T^{n,j}}\}$ and σ_* be the true value of parameter σ.

Then $\{P_{\sigma,n}\}_{\sigma,n}$ is said to be Local asymptotic mixed normality (LAMN) at $\sigma = \sigma_*$ if there exist a sequence $\{b_n\}_{n \in \mathbb{N}}$ of positive numbers, $d \times d$ symmetric positive definite random matrices Γ_n, Γ and d-dimensional random vectors $\mathcal{N}_n, \mathcal{N}$ such that $b_n \to \infty$,

$$\mathcal{L}(\mathcal{N}_n, \Gamma_n) \to^d \mathcal{L}(\mathcal{N}, \Gamma),$$

and

$$\log \frac{dP_{\sigma_* + b_n^{-1/2}u, n}}{dP_{\sigma_*, n}} - \left(u^\star \sqrt{\Gamma_n} \mathcal{N}_n - \frac{1}{2} u^\star \Gamma_n u \right) \to 0$$

in $P_{\sigma_*,n}$-probability as $n \to \infty$ for any $u \in \mathbb{R}^d$.

Jeganathan (1983) gave lower bounds of risk functions for families of probability measures which satisfy the LAMN properties by using Γ in the above equation. On the other hand, Gobet (2001) proved the LAMN property for family of probability measures generated by synchronous equi-spaced sampling $\{Y_{kT/n}\}_{k=0}^n$ of process $\{Y_t\}$ which satisfies (1).

In this work, we study the LAMN property for $\{P_{\sigma,n}\}_{\sigma,n}$ generated by nonsynchronously observed diffusion processes. To prove the LAMN property, we use Malliavin calculus approach and the quasi-log-likelihood function proposed by Ogihara and Yoshida (2012).