Nondegeneracy of statistical random field and statistics for stochastic processes

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Abstract

The Quasi likelihood analysis (QLA) is a systematic analysis of the quasi likelihood random field and the associated estimators (quasi MLE, quasi Bayesian estimators), with a large deviation method that derives precise tail probability estimates for the random field and estimators. One needs these estimates when developing the very basic fields such as asymptotic decision theory, prediction, information criteria, asymptotic expansion, higher-order inference etc. for stochastic processes.

The framework of the likelihood analysis was established by Ibragimov and Hasminskii (TPA1972, TPA1973, Springer1981) and extended to semimartingales by Kutoyants (Heldermann1984, Springer1998, Springer2004).

A polynomial type large deviation inequality was provided in Yoshida (ISMRM2006) for a quasi likelihood random field that is locally asymptotically quadratic. Since it assumes only convergences of moments of random variables like score and observed information, this method fits for nonlinear, even non-Markovian, stochastic processes.

For sampled stochastic processes, the QLA was developed for an ergodic diffusion process (Yoshida (ISMRM2006, AISM2011)) for an ergodic jump-diffusion process (Ogihara and Yoshida (SISP2011)), and for various adaptive schemes for ergodic diffusions (Uchida and Yoshida (SPA2012)).

Non-ergodic statistics appears in volatility estimation based on high-frequent data under the finite time horizon. In order to prove the polynomial type large deviation inequality, one needs the nondegeneracy of an index that expresses the degree of separation of statistical models, that is, the inequality

$$P\left[\chi_0 < \frac{1}{r}\right] \quad \leq \quad \frac{C}{r^L}(r > 0)$$

where

$$\chi_0 \quad = \quad \inf_{\theta\in\Theta} \int_0^T f(X_t, \theta)dt,$$

for a stochastic process $X = (X_t)_{t\in[0,T]}$ and a function f of (x, θ). The key index χ_0 is random, differently from ergodic cases, and this causes a problem involving a new probabilistic argument.

Uchida and Yoshida (SAPSVII 2009, ISMRM2011) provided an analytic criterion and proved that the nondegeneracy of a certain tensor field solves the problem.

We discuss a geometric criterion that is applicable even when the null set of $f(\cdot, \theta)$ is not a regular manifold (Uchida and Yoshida (arXiv2012)).

We begin with statistical background of the nondegeneracy of the statistical random field. There are applications in QLA and the higher-order asymptotic theory.