Singulary perturbed random parabolic operators
Limiting development

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Outline

Outline

Periodic structure in random media

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Model equation

Model equation

The homogenization problem for the solution of the following Cauchy problem

$$\frac{\partial u^\varepsilon}{\partial t} = \frac{\partial}{\partial x}\left(a\left(\frac{x}{\varepsilon}, \xi_{\frac{t}{\varepsilon^\alpha}}\right)\frac{\partial u^\varepsilon}{\partial x}\right),\ u^\varepsilon(0) = g(x),$$

with

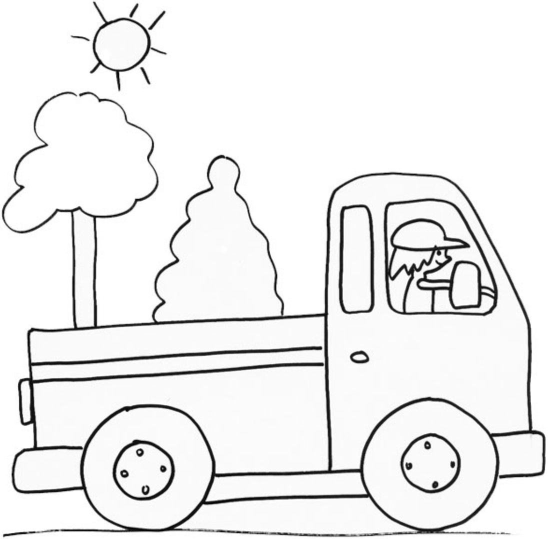
- $\alpha > 0$
- ε is a small positive parameter,
- ξ_s is a diffusion process in R^d possessing an invariant measure with density $p(y)$.
- The coefficient $a(z, y)$ is supposed to be periodic in z and uniformly elliptic.

Periodic structure in random media, II

Interpretation

We consider parabolic operators whose coefficients depend on time through some (certain) rapidly oscillating stochastic process. Such equations arise, for example, when studying the effect of random forces on microinhomogeneous medium.

Periodic structure in random media, II



Probabilistic Interpretation

Let $X_t^\varepsilon, t \in [0, T]$ be a diffusion process :

$$dX_t^\varepsilon = \sigma(\frac{X_t^\varepsilon}{\varepsilon}, \xi_{\frac{t}{\varepsilon^\alpha}})\, dW_t + \frac{1}{\varepsilon} a_z(\frac{X_t^\varepsilon}{\varepsilon}, \xi_{\frac{t}{\varepsilon^\alpha}})\, dt,\ X_0^\varepsilon = \eta$$

Then $u^\varepsilon(t, x)$ is the density of the conditional distribution of X_t^ε given by

$$\xi_{\frac{s}{\varepsilon^\alpha}}, s \leq t.$$

Difficulties

Difficulties

- there is the large parameter $\frac{1}{\varepsilon}$
- there is no convergence for the coefficients
- convergence for X^{ε} in law does not imply the convergence of conditional distributions

Two scales averaging

Crucial point

- the ratio $\frac{X_t^\varepsilon}{\varepsilon} = Z_{\frac{t}{\varepsilon^2}}$ (in law), where Z is a stationary ergodic process on the torus with the Lebesgue measure dx as an invariant measure.
- for the pair (Z, ξ) the invariant measure is the product of $dx \times \rho(y)dy$
- there are **three** cases $\alpha < 2$, $\alpha = 2$, $\alpha > 2$.

The central limit theorem scale

- For Z the rate of convergence of $\int_0^t f(s, Z_{\frac{s}{\varepsilon^2}})\, ds$ is order ε,
- For ξ —- is is order $\varepsilon^{\frac{\alpha}{2}}$,
- What happened for u^ε?

There are a lot of crucial points, I

$\alpha = 2$

The law of

$$\frac{u^\varepsilon - u^0}{\varepsilon}$$

converges to the solution of some SPDE with constant coefficients.

$$dV = [\hat{a} V_{xx} + \mu u^0_{xxx}]\, dt + \Lambda u^0_{xx}\, dW_t, \ V(0) = 0$$

There are a lot of crucial points, II

$\alpha < 2$

$$\alpha_m = \frac{2}{1 + \frac{1}{2m}}$$

$\delta = 2 - \alpha$, $n_0 = [\frac{\alpha}{2\delta}]$ The law of

$$\frac{u^\varepsilon - \Sigma_{k=0}^{n_0} \varepsilon^{k\delta} u^k}{\varepsilon^{\frac{\alpha}{2}}}$$

converges to the solution of some SPDE with constant coefficients.

$$dV = \hat{a} V_{xx}\, dt + \Lambda u^0_{xx}\, dW_t \; V(0) = 0$$

There are a lot of crucial points, III

$\alpha > 2$

$$\alpha_r = 2 + \frac{1}{r};\ \alpha_k = 2k$$

$\delta = \alpha - 2, n_0 = [\frac{1}{\delta}], N_0 = [\frac{\alpha}{2}]$
The law of

$$\frac{u^\varepsilon - \Sigma_{k=0}^{n_0} \varepsilon^{k\delta} u^k - \Sigma_{r=1}^{N_0} \varepsilon^r U^r}{\varepsilon^{\frac{\alpha}{2}}}$$

converges to the solution of some SPDE with constant coefficients.

$$dV = \hat{a} V_{xx}\, dt + \Lambda u^0_{xx}\, dW_t\ V(0) = 0$$

Limiting representation,I

$\alpha = 2$

$$u^\varepsilon = u^0 + \varepsilon\chi(\frac{x}{\varepsilon}, \frac{t}{\varepsilon^2})u^0_x + \varepsilon v^\varepsilon,$$

with an unique periodic stationary solution χ:

$$\frac{\partial\chi}{\partial t} = \frac{\partial}{\partial z}(a(z,\xi_t)\frac{\partial\chi}{\partial z}) + a_z(z,\xi_t)$$

Limiting representation, II

Let r^ε satisfy

$$\frac{\partial r^\varepsilon}{\partial t} = \frac{\partial}{\partial x}(a(\frac{x}{\varepsilon}, \xi_{\frac{t}{\varepsilon^\alpha}})\frac{\partial r^\varepsilon}{\partial x}) + f(x, t, \frac{x}{\varepsilon}, \xi_{\frac{t}{\varepsilon^\alpha}}), \ r^\varepsilon(0) = 0,$$

with f: $\overline{< f(x, t, \cdot, \cdot) >} = 0$.
Then r^ε goes to 0 as ε goes to 0.

Open questions

- Generalization for non diffusion type process
- The explicit formula for averaged coefficients
- The probabilistic interpretation of the corrector χ